



Zagazig University
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Physics of SOUND & HEAT

**For
First Year Students**

**By
Physics Department**

CHAPTER 1

Thermal Physics

How can trapped water blow off the top of a volcano in a giant explosion? What causes a sidewalk or road to fracture and buckle spontaneously when the temperature changes? How can thermal energy be harnessed to do work, running the engines that make everything in modern living possible?

Answering these and related questions is the domain of thermal physics, the study of temperature, heat, and how they affect matter. Quantitative descriptions of thermal phenomena require careful definitions of the concepts of temperature, heat, and internal energy. Heat leads to changes in internal energy and thus to changes in temperature, which cause the expansion or contraction of matter. Such changes can damage roadways and buildings, create stress fractures in metal, and render flexible materials stiff and brittle, the latter resulting in compromised O-

rings and the *Challenger* disaster. Changes in internal energy can also be harnessed for transportation, construction, and food preservation.

Gases are critical in the harnessing of thermal energy to do work. Within normal temperature ranges, a gas acts like a large collection of non-interacting point particles, called an ideal gas. Such gases can be studied on either a macroscopic or microscopic scale. On the macroscopic scale, the pressure, volume, temperature, and number of particles associated with a gas can be related in a single equation known as the ideal gas law. On the microscopic scale, a model called the kinetic theory of gases pictures the components of a gas as small particles. This model will enable us to understand how processes on the atomic scale affect macroscopic properties like pressure, temperature, and internal energy.

1.1 TEMPERATURE AND THE ZEROth LAW OF THERMODYNAMICS

Temperature is commonly associated with how hot or cold an object feels when we touch it. While our senses provide us with qualitative indications of temperature, they are unreliable and often misleading. A metal ice tray feels colder to the hand, for example, than a package of frozen vegetables at the same temperature, due to the fact metals conduct thermal energy more rapidly than a cardboard package.

What we need is a reliable and reproducible method of making quantitative measurements that establish the relative “hotness” or “coldness” of objects—a method related solely to temperature. Scientists have developed a variety of thermometers for making such measurements.

When placed in contact with each other, two objects at different initial temperatures will eventually reach a common intermediate temperature. If a cup of hot coffee is cooled with an ice cube, for example, the ice rises in

temperature and eventually melts while the temperature of the coffee decreases. Understanding the concept of temperature requires understanding *thermal contact* and *thermal equilibrium*. Two objects are in thermal contact if energy can be exchanged between them. Two objects are in thermal equilibrium if they are in thermal contact and there is no net exchange of energy.

The exchange of energy between two objects because of differences in their temperatures is called heat, a concept examined in more detail in the next Chapter.

Using these ideas, we can develop a formal definition of temperature. Consider two objects A and B that are not in thermal contact with each other, and a third object C that acts as a thermometer—a device calibrated to measure the temperature of an object. We wish to determine whether A and B would be in thermal equilibrium if they were placed in thermal contact. The thermometer (object C) is first placed in thermal contact with A until thermal equilibrium is reached, as in Figure

1.1a, whereupon the reading of the thermometer is recorded. The thermometer is then placed in thermal contact with B, and its reading is again recorded at equilibrium Fig. 1.1b. If the two readings are the same, then A and B are in thermal equilibrium with each other. If A and B are placed in thermal contact with each other, as in Figure 1.1c, there is no net transfer of energy between them.

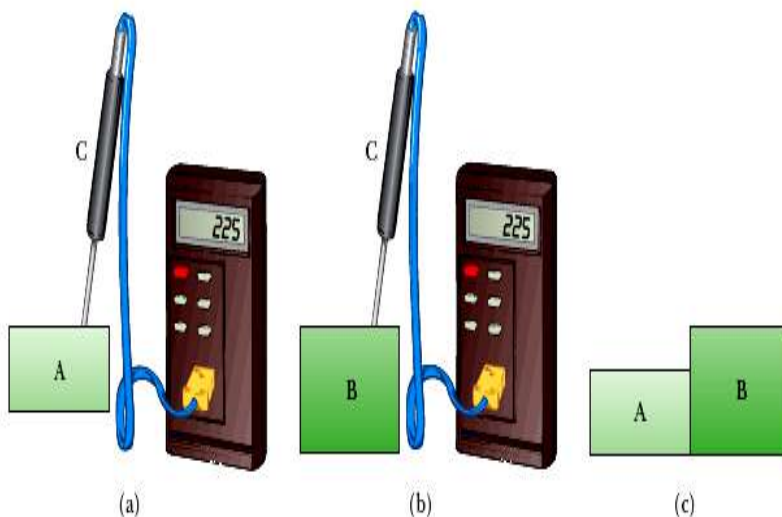


Figure 1.1 The zeroth law of thermodynamics. (a) and (b): If the temperatures of A and B are found to be the same as measured by object C (a thermometer), no energy will be exchanged between them when they are placed in thermal contact with each other, as in (c).

We can summarize these results in a statement known as the zeroth law of thermodynamics (the law of equilibrium):

Zeroth law of thermodynamics states that: If objects A and B are separately in thermal equilibrium with a third object C, then A and B are in thermal equilibrium with each other.

This statement is important because it makes it possible to define *temperature*. We can think of temperature as the property that determines whether or not an object is in thermal equilibrium with other objects. *Two objects in thermal equilibrium with each other are at the same temperature*

Quick Quiz

Two objects with different sizes, masses, and temperatures are placed in thermal contact. Choose the best answer: Energy travels (a) from the larger object to the smaller object (b) from the object with more mass to the one with less mass (c) from the object at higher temperature to the object at lower temperature..

1.2 THERMOMETERS AND TEMPERATURE SCALES

Thermometers are devices used to measure the temperature of an object or a system. When a thermometer is in thermal contact with a system, energy is exchanged until the thermometer and the system are in thermal equilibrium with each other. For accurate readings, the thermometer must be much smaller than the system, so that the energy the thermometer gains or loses doesn't significantly alter the energy content of the system.

All thermometers make use of some physical property that changes with temperature and can be calibrated to make the temperature measurable. Some of the physical properties used are:

- (1) the volume of a liquid,
- (2) the length of a solid,
- (3) the pressure of a gas held at constant volume,
- (4) the volume of a gas held at constant pressure,
- (5) the electric resistance of a conductor, and
- (6) the color of a very hot object.

One common thermometer in everyday use consists of a mass of liquid—usually mercury or alcohol—that expands into a glass capillary tube when its temperature rises (Fig. 1.2). In this case the physical property that changes is the volume of a liquid. To serve as an effective thermometer, the change in volume of the liquid with change in temperature must be very nearly constant over the temperature ranges of interest. When the cross-sectional area of the capillary tube is constant as well, the change in volume of the liquid varies linearly with its length along the tube. We can then define a temperature in terms of the length of the liquid column.

The thermometer can be calibrated by placing it in thermal contact with environments that remain at constant temperature. One such environment is a mixture of water and ice in thermal equilibrium at atmospheric pressure. Another commonly used system is a mixture of water and steam in thermal equilibrium at atmospheric pressure.

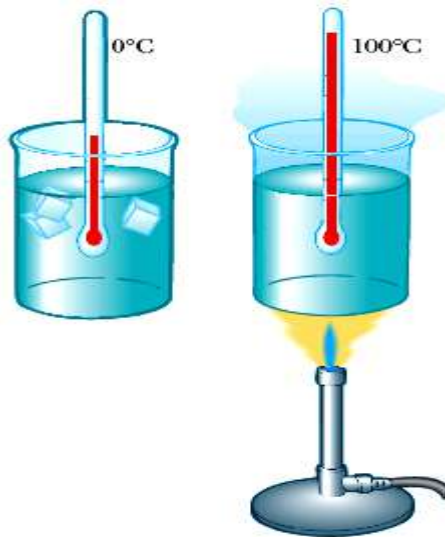


Figure 1.2 Schematic diagram of a mercury thermometer. Because of thermal expansion, the level of the mercury rises as the temperature of the mercury changes from 0°C (the ice point) to 100°C (the steam point).

Once we have marked the ends of the liquid column for our chosen environment on our thermometer, we need to define a scale of numbers associated with various temperatures.

An example of such a scale is the *Celsius temperature scale*, formerly called the centigrade scale. On the Celsius scale, the temperature of the ice–water mixture is defined to be zero degrees Celsius, written 0°C and called the *ice point* or *freezing point* of water.

The temperature of the water–steam mixture is defined as 100°C , called the *steam point* or *boiling point* of water. Once the ends of the liquid column in the thermometer have been marked at these two points, the distance between marks is divided into 100 equal segments, each corresponding to a change in temperature of one degree Celsius. Thermometers calibrated in this way present problems when extremely accurate readings are needed. For example, an alcohol thermometer calibrated at the ice and steam points of water might agree with a mercury thermometer only at the calibration points. Because mercury and alcohol have different thermal expansion properties, when one indicates a temperature of 50°C , say, the other may indicate a slightly different temperature. The discrepancies between different types of thermometers are especially large when the temperatures to be measured are far from the calibration points

The Constant-Volume Gas Thermometer and the Kelvin Scale

We can construct practical thermometers such as the mercury thermometer, but these types of thermometers don't define temperature in a fundamental way. One thermometer, however, is more fundamental, and offers a way to define temperature and relate it directly to internal energy: the *gas thermometer*. In a gas thermometer, the temperature readings are nearly independent of the substance used in the thermometer. One type of gas thermometer is the constant-volume unit shown in Figure 1.3.

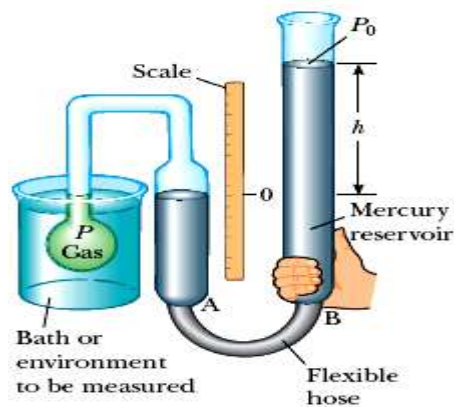


Figure 1.3 A constant-volume gas thermometer measures the pressure of the gas contained in the flask immersed in the bath. The volume of gas in the flask is kept constant by raising or lowering reservoir B to keep the mercury level constant.

The behavior observed in this device is the variation of pressure with temperature of a fixed volume of gas. When the constant-volume gas thermometer was developed, it was calibrated using the ice and steam points of water as follows (a different calibration procedure, to be discussed shortly, is now used): The gas flask is inserted into an ice-water bath, and mercury reservoir B is raised or lowered until the volume of the confined gas is at some value, indicated by the zero point on the scale. The height h , the difference between the levels in the reservoir and column A, indicates the pressure in the flask at 0°C . The flask is inserted into water at the steam point, and reservoir B is readjusted until the height in column A is again brought to zero on the scale, ensuring that the gas volume is the same as it had been in the ice bath (hence the designation "constant-volume"). A measure of the new value for h gives a value for the pressure at 100°C . These pressure and temperature values are then plotted on a graph, as in Figure 1.4. The line connecting the two points serves as a calibration curve for

measuring unknown temperatures. If we want to measure the temperature of a substance, we place the gas flask in thermal contact with the substance and adjust the column of mercury until the level in column A returns to zero.

The height of the mercury column tells us the pressure of the gas, and we could then find the temperature of the substance from the calibration curve.

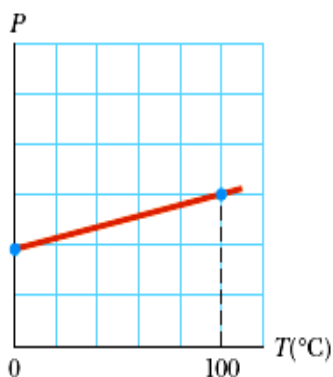


Figure 1.4 A typical graph of

pressure versus temperature taken with a constant-volume gas thermometer. The dots represent known reference temperatures (the ice and the steam points of water).

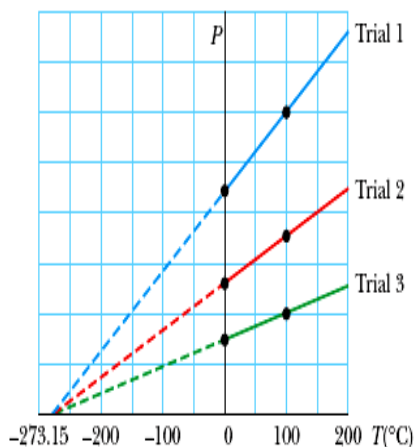


Figure1.5

Pressure versus temperature

for experimental trials in which gases have different pressures in a constant-volume gas thermometer. Note that for all three trials the pressure extrapolates to zero at the temperature -273.15°C .

Now suppose that temperatures are measured with various gas thermometers containing different gases. Experiments show that the thermometer readings are nearly independent of the type of gas used, as long as the gas pressure is low and the temperature is well above the point at which the gas liquifies. We can also perform the temperature measurements with the gas in the flask at different starting pressures at 0°C. As long as the pressure is low, we will generate straight-line calibration curves for each starting pressure, as shown for three experimental trials (solid lines) in Figure 1.5.

If the curves in (Figure 1.5) are extended back toward negative temperatures, we find a startling result: In every case, regardless of the type of gas or the value of the low starting pressure, the pressure extrapolates to zero when the temperature is -273.15°C. This fact suggests that this particular temperature is universal in its importance, because it doesn't depend on the substance used in the thermometer. In addition, because the lowest possible pressure is $P = 0$, a perfect vacuum, the

temperature -273.15°C must represent a lower bound for physical processes. We define this temperature as absolute zero.

Absolute zero is used as the basis for the Kelvin temperature scale, which sets -273.15°C as its zero point (0 K). The size of a "degree" on the Kelvin scale is chosen to be identical to the size of a degree on the Celsius scale. The relationship between these two temperature scales is

$$T_C = T - 273.15 \quad [1.1]$$

Where T_C is the Celsius temperature and T is the Kelvin temperature (sometimes called the absolute temperature).

Technically, Equation 1.1 should have units on the right-hand side so that it reads $T_C = T \frac{^{\circ}\text{C}}{\text{K}} - 273.15 \text{ } ^{\circ}\text{C}$. The units are rather cumbersome in this context, so we will usually suppress them in such calculations except in the final answer. (This will also be the case when discussing the Celsius and Fahrenheit scales).

Early gas thermometers made use of ice and steam points according to the procedure just described. These points are experimentally difficult to duplicate, however, because they are pressure-sensitive. Consequently, a procedure based on two new points was adopted in 1954 by the International Committee on Weights and Measures. The first point is absolute zero. The second point is the triple point of water, which is the single temperature and pressure at which water, water vapor, and ice can coexist in equilibrium. This point is a convenient and reproducible reference temperature for the Kelvin scale; it occurs at a temperature of 0.01°C and a pressure of 4.58 mm of mercury. The temperature at the triple point of water on the Kelvin scale occurs at 273.16 K. Therefore, the SI unit of temperature,

The kelvin, is defined as $1/273.16$ of the temperature of the triple point of water. Figure 1.6 shows the Kelvin temperatures for various physical processes and structures. Absolute zero has been closely approached but never achieved.

What would happen to a substance if its temperature could reach 0 K? As Figure 1.5 indicates, the substance would exert zero pressure on the walls of its container (assuming that the gas doesn't liquefy or solidify on the way to absolute zero). In Section 1.5 we show that the pressure of a gas is proportional to the kinetic energy of the molecules of that gas. According to classical physics, therefore, the kinetic energy of the gas would go to zero, and there would be no motion at all of the individual components of the gas. According to quantum theory, however the gas would always retain some residual energy, called the *zero-point energy*, at that low temperature.

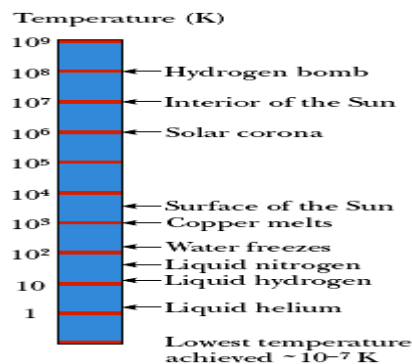


Figure 1.6 Absolute temperatures at which various selected physical processes take place. Note that the scale is logarithmic.

The Celsius, Kelvin, and Fahrenheit Temperature Scales

Equation 1.1 shows that the Celsius temperature T_C is shifted from the absolute (Kelvin) temperature T by 273.15. Because the size of a Celsius degree is the same as a kelvin, a temperature difference of 5°C is equal to a temperature difference of 5 K. The two scales differ only in the choice of zero point. The ice point (273.15 K) corresponds to 0.00°C , and the steam point (373.15 K) is equivalent to 100.00°C .

The most common temperature scale in use in the United States is the Fahrenheit scale. It sets the temperature of the ice point at 32°F and the temperature of the steam point at 212°F . The relationship between the Celsius and Fahrenheit temperature scales is

$$T_F = \frac{9}{5} T_C + 32 \quad [1.2a]$$

For example, a temperature of 50.0°F corresponds to a Celsius temperature of 10.0°C and an absolute temperature of 283 K.

Equation 1.2 can be inverted to give Celsius temperatures in terms of Fahrenheit temperatures:

$$T_C = \frac{5}{9} (T_F - 32) \quad [1.2b]$$

Equation 1.2 can also be used to find a relationship between changes in temperature on the Celsius and Fahrenheit scales. In a problem at the end of the chapter you will be asked to show that if the Celsius temperature changes by ΔT_C , the Fahrenheit temperature changes by the amount

$$\Delta T_F = \frac{9}{5} \Delta T_C \quad [1.3]$$

Figure 1.7 compares the three temperature scales we have discussed.

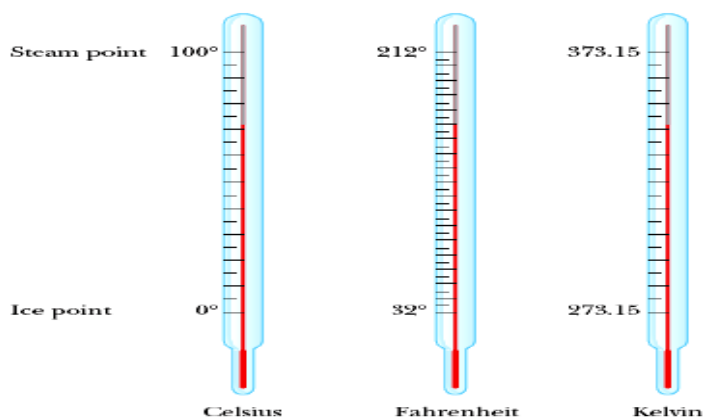


Figure 1.7 A comparison of the Celsius, Fahrenheit, and Kelvin temperature scales.

EXAMPLE 1.1 Skin Temperature

The temperature gradient between the skin and the air is regulated by cutaneous (skin) blood flow. If the cutaneous blood vessels are constricted, the skin temperature and the temperature of the environment will be about the same. When the vessels are dilated, more blood is brought to the surface. Suppose that during dilation the skin warms from 72.0°F to 84.0°F . (a) Convert these temperatures to Celsius and find the difference. (b) Convert the temperatures to Kelvin, again finding the difference.

Solution

(a) Convert the temperatures from Fahrenheit to Celsius and find the difference.

$$T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(72 - 32) = 22^{\circ}\text{C}$$

$$T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(84 - 32) = 29^{\circ}\text{C}$$

$$\Delta T_C = 29^{\circ}\text{C} - 22^{\circ}\text{C} = 7^{\circ}\text{C}$$

(b) Convert the temperatures from Fahrenheit to Kelvin and find their difference.

$$T_C = T - 273.15 \rightarrow T = T_C + 273.15$$

$$T = 22 + 273.15 = 285 \text{ K}$$

$$T = 29 + 273.15 = 292 \text{ K}$$

$$\Delta T = 292 \text{ K} - 285 \text{ K} = 7 \text{ K}$$

Remark The change in temperature in Kelvin and Celsius is the same, as it should be.

EXAMPLE 1.2

Extraterrestrial Temperature Scale

Goal Understand how to relate different temperature scales.

Problem An extraterrestrial scientist invents a temperature scale such that water freezes at -75°E and boils at 325°E , where E stands for an extraterrestrial scale. Find an equation that relates temperature in $^\circ\text{E}$ to temperature in $^\circ\text{C}$.

Solution

Find the change in temperature in $^\circ\text{E}$ between the freezing and boiling points of water:

$$\Delta T_E = 325^\circ\text{E} - (-75^\circ\text{E}) = 400^\circ\text{E}$$

Find the change in temperature in $^{\circ}\text{C}$ between the freezing and boiling points of water:

$$\Delta T_C = 100^{\circ}\text{C} - 0^{\circ}\text{C} = 100^{\circ}\text{C}$$

Form the ratio of these two quantities.

$$\frac{\Delta T_E}{\Delta T_C} = \frac{400^{\circ}\text{E}}{100^{\circ}\text{C}} = 4 \frac{^{\circ}\text{E}}{^{\circ}\text{C}}$$

This ratio is the same between any other two temperatures

—say, from the freezing point to an unknown final temperature.

Set the two ratios equal to each other:

$$\frac{\Delta T_E}{\Delta T_C} = \frac{T_E - (-75^{\circ}\text{E})}{T_C - 0^{\circ}\text{C}} = 4 \frac{^{\circ}\text{E}}{^{\circ}\text{C}}$$

Solve for T_E :

$$T_E - (-75^{\circ}\text{E}) = 4(^{\circ}\text{E}/^{\circ}\text{C})(T_C - 0^{\circ}\text{C})$$

$$T_E = 4T_C - 75$$

Remark The relationship between any other two temperatures scales can be derived in the same way.

1.3 THERMAL EXPANSION OF SOLIDS AND LIQUIDS

Our discussion of the liquid thermometer made use of one of the best-known changes that occur in most substances: As temperature of the substance increases, its volume increases. This phenomenon, known as thermal expansion, plays an important role in numerous applications. Thermal expansion joints, for example, must be included in buildings, concrete highways, and bridges to compensate for changes in dimensions with variations in temperature.

The overall thermal expansion of an object is a consequence of the change in the average separation between its constituent atoms or molecules.

To understand this idea, consider how the atoms in a solid substance behave. These atoms are located at fixed equilibrium positions; if an atom is pulled away from its position, Figure 1.8, a restoring force pulls it back. We can imagine that the atoms are particles connected by springs to their neighboring atoms. If an atom is pulled away

from its equilibrium position, the distortion of the springs provides a restoring force.

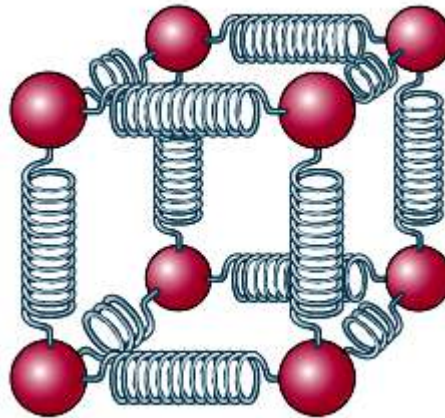


Figure 1.8 A model of a portion of a solid. The atoms (spheres) are imagined as being attached to each other by springs, which represent the elastic nature of the interatomic forces. A solid consists of trillions of segments like this, with springs connecting all of them.

At ordinary temperatures, the atoms vibrate around their equilibrium positions with an amplitude (maximum distance from the center of vibration) of about 10^{-11}m , with an average spacing between the atoms of about 10^{-10}m . As the temperature of the solid increases, the atoms vibrate with greater

amplitudes and the average separation between them increases. Consequently, the solid as a whole expands.

If the thermal expansion of an object is sufficiently small compared with the object's initial dimensions, then the change in any dimension is, to a good approximation, proportional to the first power of the temperature change. Suppose an object has an initial length L_0 along some direction at some temperature T_0 . Then the length increases by ΔL for a change in temperature ΔT . So for small changes in temperature,

$$\Delta L = \alpha L_0 \Delta T \quad [1.4]$$

Or

$$L - L_0 = \alpha L_0 (T - T_0)$$

where L is the object's final length T is its final temperature, and the proportionality constant α is called the coefficient of linear expansion for a given material and has units of $(^\circ\text{C})^{-1}$.

Table 1.1 lists the coefficients of linear expansion for various materials. Note that for these materials α is positive, indicating an increase in length with increasing temperature.

Thermal expansion affects the choice of glassware used in kitchens and laboratories. If hot liquid is poured into a cold container made of ordinary glass, the container may well break due to thermal stress. The inside surface of the glass becomes hot and expands, while the outside surface is at room temperature, and ordinary glass may not withstand the difference in expansion without breaking. Pyrex[®] glass has a coefficient of linear expansion of about one-third that of ordinary glass, so the thermal stresses are smaller. Kitchen measuring cups and laboratory beakers are often made of Pyrex so they can be used with hot liquids.

TABLE 1.1

Average Coefficients of Expansion for Some Materials Near Room Temperature			
Material	Average Coefficient of Linear Expansion [(°C) ⁻¹]	Material	Average Coefficient of Volume Expansion [(°C) ⁻¹]
Aluminum	24×10^{-6}	Ethyl alcohol	1.12×10^{-4}
Brass and bronze	19×10^{-6}	Benzene	1.24×10^{-4}
Copper	17×10^{-6}	Acetone	1.5×10^{-4}
Glass (ordinary)	9×10^{-6}	Glycerin	4.85×10^{-4}
Glass (Pyrex [®])	3.2×10^{-6}	Mercury	1.82×10^{-4}
Lead	29×10^{-6}	Turpentine	9.0×10^{-4}
Steel	11×10^{-6}	Gasoline	9.6×10^{-4}
Invar (Ni-Fe alloy)	0.9×10^{-6}	Air	3.67×10^{-3}
Concrete	12×10^{-6}	Helium	3.665×10^{-3}

EXAMPLE 1.3 Expansion of a Railroad Track

Goal Apply the concept of linear expansion and relate it to stress.

Problem: A steel railroad track has a length of 30.000 m when the temperature is 0 °C.

(a) What is its length on a hot day when the temperature is 40.0 °C?

(b) What is the length of the same railroad track on a cold winter day when the temperature is 0 °F?

Solution

(a) Find the length of the track at 40.0 °C. Substitute given quantities into Equation 1.4, finding the change in length:

$$\begin{aligned}\Delta L &= \alpha L_0 \Delta T = [11 \times 10^{-6} (\text{°C})^{-1}] (30.000 \text{ m}) (40.0 \text{ °C}) \\ &= 0.013 \text{ m}\end{aligned}$$

Add the change to the original length to find the final length

$$L = L_0 + \Delta L = 30.013 \text{ m}$$

(b) With the same procedures the Answer will be 29.994 m.

Bimetallic Strips and Thermostats

How can different coefficients of expansion for metals be used as a temperature gauge and control electronic devices such as air conditioners?

Explanation When the temperatures of a brass rod and a steel rod of equal length are raised by the same amount from some common initial value, the brass rod expands more than the steel rod because brass has a larger coefficient of expansion than steel.

A simple device that uses this principle is a bimetallic strip. Such strips can be found in the thermostats of certain home heating systems. The strip is made by securely bonding two different metals together. As the temperature of the strip increases, the two metals expand by different amounts and the strip bends, as in Figure 1.9. The change in shape can make or break an electrical connection

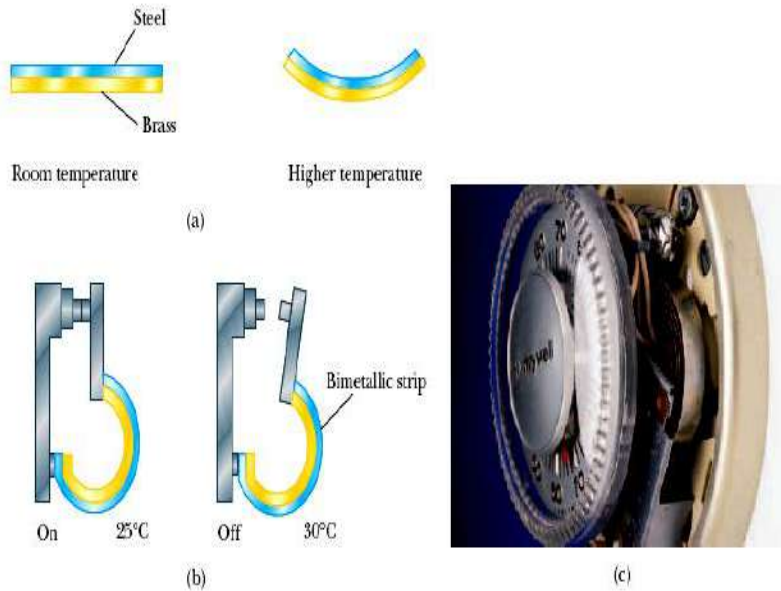
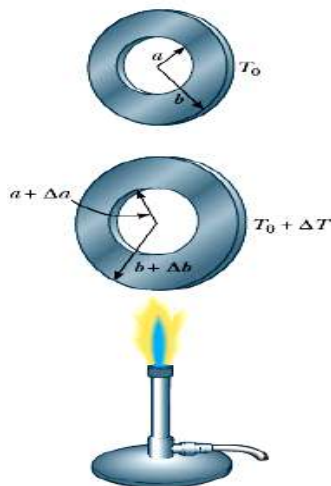


Figure 1.9 (a) A bimetallic strip bends as the temperature changes because the two metals have different coefficients of expansion. (b) A bimetallic strip used in a thermostat to break or make electrical contact. (c) The interior of a thermostat, showing the coiled bimetallic strip.

It may be helpful to picture a thermal expansion as a magnification or a photographic enlargement. For example, as the temperature of a metal washer increases (Active Fig. 1.10), all dimensions, including the radius of the hole, increase according to Equation 10.4.



ACTIVE FIGURE 1.10 Thermal expansion of a homogeneous metal washer. As the washer is heated, all dimensions increase

One practical application of thermal expansion is the common technique of using hot water to loosen a metal lid stuck on a glass jar. This works because the circumference of the lid expands more than the rim of the jar.

Because the linear dimensions of an object change due to variations in temperature, it follows that surface area and volume of the object also change. Consider a square of material having an initial length L_0 on a side and therefore an initial area $A_0 =$

L_0^2 . As the temperature is increased, the length of each side increases to.

$$L = L_0 + \alpha L_0 \Delta T$$

The new area A is

$$A = L^2 = (L_0 + \alpha L_0 \Delta T)(L_0 + \alpha L_0 \Delta T) = L_0^2 + 2\alpha L_0^2 \Delta T + \alpha^2 L_0^2 (\Delta T)^2$$

The last term in this expression contains the quantity $\alpha \Delta T$ raised to the second power. Because $\alpha \Delta T$ is much less than one, squaring it makes it even smaller. Consequently, we can neglect this term to get a simpler expression:

$$A = L_0^2 + 2\alpha L_0^2 \Delta T$$

$$A = A_0 + 2\alpha A_0 \Delta T$$

so that

$$\Delta A = A - A_0 = \gamma A_0 \Delta T \quad [1.5]$$

where $\gamma = 2\alpha$. The quantity γ is called the coefficient of area expansion.

EXAMPLE 1.4 Rings and Rods

Goal Apply the equation of area expansion.

Problem (a) A circular copper ring at 20.0°C has a hole with an area of 9.98 cm^2 . What minimum temperature must it have so that it

can be slipped onto a steel metal rod having a cross-sectional area of 10.0 cm^2 ? (b) Suppose the ring and the rod are heated simultaneously. What change in temperature of both will allow the ring to be slipped onto the end of the rod? (Assume no significant change in the coefficients of linear expansion over this temperature range.)

Solution

(a) Find the temperature of the ring that will allow it to slip onto the rod.

Substitute the desired change in area into Equation 1.5, finding the necessary change in temperature:

$$\begin{aligned}\Delta A &= \gamma A_0 \Delta T = [34 \times 10^{-6} (\text{°C})^{-1}](9.98 \text{ cm}^2)(\Delta T) \\ &= 0.02 \text{ cm}^2\end{aligned}$$

Solve for ΔT , then add this change to the initial temperature to get the final temperature:

$$\Delta T = 58.9^\circ\text{C}$$

$$T = T_0 + \Delta T = 20.0^\circ\text{C} + 58.9^\circ\text{C} = 78.9^\circ\text{C}$$

(b) Increase the temperature of both ring and rod to remove the ring.

Set the final areas of the copper ring and steel rod equal to each other:

$$A_C + \Delta A_C = A_S + \Delta A_S$$

Substitute for each change in area, ΔA :

Rearrange terms to get ΔT on one side only, factor it out and solve

$$A_C + \gamma_C A_C \Delta T = A_S + \gamma_S A_S \Delta T$$

$$\gamma_C A_C \Delta T - \gamma_S A_S \Delta T = A_S - A_C$$

$$(\gamma_C A_C - \gamma_S A_S) \Delta T = A_S - A_C$$

$$\Delta T = \frac{A_S - A_C}{\gamma_C A_C - \gamma_S A_S}$$

$$= \frac{10.0 \text{ cm}^2 - 9.98 \text{ cm}^2}{(34 \times 10^{-6} \text{ }^\circ\text{C}^{-1})(9.98 \text{ cm}^2) - (22 \times 10^{-6} \text{ }^\circ\text{C}^{-1})(10.0 \text{ cm}^2)}$$

$$\Delta T = 168^\circ\text{C}$$

We can also show that the *increase in volume* of an object accompanying a change in temperature is

$$\Delta V = \beta V_0 \Delta T \quad [1.6]$$

where β , the coefficient of volume expansion, is equal to 3α . (Note that $\gamma = 2\alpha$ and $\beta = 3\alpha$ only if the coefficient of linear expansion of the object is the same in all directions.) The proof of Equation 1.6 is similar to the proof of Equation 1.5.

As Table 1.1 indicates, each substance has its own characteristic coefficients of expansion.

The thermal expansion of water has a profound influence on rising ocean levels. At current rates of global warming, scientists predict that about one-half of the expected rise in sea level will be caused by thermal expansion; the remainder will be due to the melting of polar ice.

EXAMPLE 1.5 Global Warming and Coastal Flooding

Goal Apply the volume expansion equation together with linear expansion.

Problem (a) Estimate the fractional change in the volume of Earth's oceans due to an average temperature change of 1°C . (b) Use the fact that the average depth of the ocean is $4.00 \times 10^3 \text{ m}$ to estimate the change in depth. Note that $\beta_{\text{water}} = 2.07 \times 10^{-4} (^{\circ}\text{C})^{-1}$ level only slightly.

Solution

(a) Find the fractional change in volume.

$$\Delta V = \beta V_0 \Delta T$$

Divide the volume expansion equation by V_0 and substitute:

$$\left(\frac{\Delta V}{V_0}\right) = \beta \Delta T = (2.07 \times 10^{-4} (\text{°C})^{-1}) \cdot (1^\circ\text{C}) = 2 \times 10^{-4}$$

(b) Find the approximate increase in depth.

$$\Delta L = \alpha L_0 \Delta T = \left(\frac{\beta}{3}\right) L_0 \Delta T$$

Use the linear expansion equation. Divide the volume expansion coefficient of water by three to get the equivalent linear expansion coefficient:

$$\Delta L = (6.90 \times 10^{-5} (\text{°C})^{-1}) (4\,000 \text{ m}) (1^\circ\text{C}) \approx 0.3 \text{ m}$$

The Unusual Behavior of Water

Liquids generally increase in volume with increasing temperature and have volume expansion coefficients about ten times greater than those of solids. Over a small temperature range, water is an exception to this rule, as we can see from its density-versus-temperature curve in Figure 1.11. As the temperature increases from 0°C to 4°C, water contracts, so its density increases. Above 4°C, water exhibits the expected expansion with increasing temperature. The density of water reaches its maximum value of 1 000 kg/m³ at 4°C.

We can use this unusual thermal expansion behavior of water to explain why a pond freezes slowly from the top down. When the atmospheric temperature drops from 7°C to 6°C, say, the water at the surface of the pond also cools and consequently decreases in volume. This means the surface water is more dense than the water below it, which has not yet cooled nor decreased in volume. As a result, the surface water sinks and warmer water from below is forced to the surface to be cooled, a

process called *upwelling*. When the atmospheric temperature is between 4°C and 0°C, however, the surface water expands as it cools, becoming less dense than the water below it.

The sinking process stops, and eventually the surface water freezes. As the water freezes, the ice remains on the surface because ice is less dense than water. The ice continues to build up on the surface, and water near the bottom of the pool remains at 4°C. Further, the ice forms an insulating layer that slows heat loss from the underlying water, offering thermal protection for marine life.

Without buoyancy and the expansion of water upon freezing, life on Earth may not have been possible. If ice had been more dense than water, it would have sunk to the bottom of the ocean and built up over time. This could have led to a freezing of the oceans, turning the Earth into an icebound world.

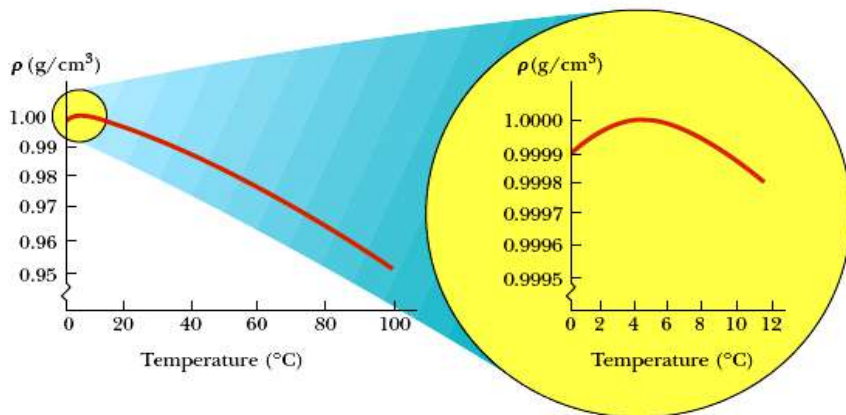


Figure 1.11 The density of water as a function of temperature. The inset at the right shows that the maximum density of water occurs at 4°C.

Bursting Pipes in Winter

The same peculiar thermal expansion properties of water sometimes cause pipes to burst in winter. As energy leaves the water through the pipe by heat and is transferred to the outside cold air, the outer layers of water in the pipe freeze first.

The continuing energy transfer causes ice to form ever closer to the center of the pipe. As long as there is still an opening through the ice, the water can expand as its temperature approaches 0°C or as it freezes into more ice, pushing itself into another part of the pipe. Eventually, however, the ice will freeze to the center somewhere along the

pipe's length, forming a plug of ice at that point. If there is still liquid water between this plug and some other obstruction, such as another ice plug or a spigot, then no additional volume is available for further expansion and freezing. The pressure in the pipe builds and can rupture the pipe.

1.4 MACROSCOPIC DESCRIPTION OF AN IDEAL GAS

The properties of gases are important in a number of thermodynamic processes. Our weather is a good example of the types of processes that depend on the behavior of gases.

If we introduce a gas into a container, it expands to fill the container uniformly, with its pressure depending on the size of the container, the temperature, and the amount of gas. A larger container results in a lower pressure, while higher temperatures or larger amounts of gas result in a higher pressure. The pressure P , volume V , temperature T , and amount n of gas in a container are related to each other by an *equation of state*.

The equation of state can be very complicated, but is found experimentally to be relatively simple if the gas is maintained at a low pressure (or a low density).

Such a low-density gas approximates what is called an ideal gas. Most gases at room temperature and atmospheric pressure behave approximately as ideal gases. An ideal gas is a collection of atoms or molecules that move randomly and exert no long-range forces on each other. Each particle of the ideal gas is individually pointlike, occupying a negligible volume.

A gas usually consists of a very large number of particles, so it's convenient to express the amount of gas in a given volume in terms of the number of moles, n . A mole is a number. The same number of particles is found in a mole of helium as in a mole of iron or aluminum. This number is known as *Avogadro's number* and is given by

$$N_A = 6.02 \times 10^{23} \quad \text{particles/mole}$$

Avogadro's number and the definition of a mole are fundamental to chemistry and related branches of physics. The number of

moles of a substance is related to its mass m by the expression

$$n = m / \text{molar mass} \quad [1.7]$$

where the molar mass of the substance is defined as the mass of one mole of that substance, usually expressed in grams per mole.

There are lots of atoms in the world, so it's natural and convenient to choose a very large number like Avogadro's number when describing collections of atoms. At the same time, Avogadro's number must be special in some way, because otherwise why not just count things in terms of some large power of ten, like 10^{24} ?

It turns out that Avogadro's number was chosen so that the mass in grams of one Avogadro's number of an element is numerically the same as the mass of one atom of the element, expressed in atomic mass units (u).

This relationship is very convenient. Looking at the periodic table of the elements in the back of the book, we find that carbon has an atomic mass of 12 u, so 12 g of

carbon consists of exactly 6.02×10^{23} atoms of carbon. The atomic mass of oxygen is 16 u, so in 16g of oxygen there are again 6.02×10^{23} atoms of oxygen.

The same holds true for molecules: The molecular mass of molecular hydrogen, H_2 , is 2 u, and there is an Avogadro's number of molecules in 2 g of molecular hydrogen.

The technical definition of a mole is as follows: One mole (mol) of any substance is that amount of the substance that contains as many particles (atoms, molecules, or other particles) as there are atoms in 12 g of the isotope carbon-12.

Taking carbon-12 as a test case, let's find the mass of an Avogadro's number of carbon-12 atoms. A carbon-12 atom has an atomic mass of 12 u, or 12 atomic mass units. One atomic mass unit is equal to 1.66×10^{-24} g, about the same as the mass of a neutron or proton—particles that make up atomic nuclei. The mass m of an Avogadro's number of carbon-12 atoms is then given by

$$m = N_A(12 \text{ u}) = 6.02 \times 10^{23}(12 \text{ u}) \left(\frac{1.66 \times 10^{-24} \text{ g}}{\text{u}} \right) = 12.0 \text{ g}$$

So we see that Avogadro's number is deliberately chosen to be the inverse of the number of grams in an atomic mass unit. In this way, the atomic mass of an atom expressed in atomic mass units is numerically the same as the mass of an Avogadro's number of that kind of atom expressed in grams. Because there are 6.02×10^{23} particles in one mole of *any* element, the mass per atom for a given element is

$$m_{\text{atom}} = \frac{\text{molar mass}}{N_A}$$

For example the mass of helium atom is

$$m_{\text{He}} = \frac{4.00 \text{ g/mol}}{6.02 \times 10^{23} \text{ atoms/mol}} = 6.64 \times 10^{-24} \text{ g/atom}$$

Now suppose an ideal gas is confined to a cylindrical container with a volume that can be changed by moving a piston, as in Active Figure 1.12. Assume that the cylinder doesn't leak, so the number of moles remains constant. Experiments yield the following observations: First, when the gas is kept at a constant temperature, its pressure is inversely proportional to its volume (Boyle's law). Second, when the pressure of the gas is kept constant, the volume of the gas is

directly proportional to the temperature (Charles's law.) Third, when the volume of the gas is held constant, the pressure is directly proportional to the temperature (Gay-Lussac's law).

These observations can be summarized by the following equation of state, known as the ideal gas law:

$$PV = nRT \quad [1.8]$$

In this equation, R is a constant for a specific gas that must be determined from experiments, while T is the temperature in kelvins. Each point on a P versus V diagram would represent a different state of the system. Experiments on several gases show that, as the pressure approaches zero, the quantity PV/nT approaches the same value of R for all gases. For this reason, R is called the universal gas constant.

In SI units, where pressure is expressed in pascals and volume in cubic meters,

$$R = 8.31 \text{ J/mol}\cdot\text{K} \quad [1.9]$$

If the pressure is expressed in atmospheres and the volume is given in liters (recall that $1 \text{ L} = 10^3 \text{ cm}^3 = 10^{-3} \text{ m}^3$), then

$$R = 0.0821 \text{ L} \cdot \text{atm} / \text{mol} \cdot \text{K}$$

Using this value of R and Equation 1.8, the volume occupied by 1 mol of any ideal gas at atmospheric pressure and at 0°C (273 K) is 22.4 L



Figure 1.12 a gas confined to a cylinder whose volume can be varied with a movable piston

EXAMPLE 1.6 An Expanding Gas

Goal Use the ideal gas law to analyze a system of gas.

Problem An ideal gas at 20.0°C and a pressure of $1.50 \times 10^5 \text{ Pa}$ is in a container having a volume of 1.00 L. (a) Determine the

number of moles of gas in the container. (b) The gas pushes against a piston, expanding to twice its original volume, while the pressure falls to atmospheric pressure. Find the final temperature.

Solution

(a) Find the number of moles of gas.

Convert the temperature to kelvins:

$$T = T_C + 273 = 20.0 + 273 = 293 \text{ K}$$

Solve the ideal gas law for n and substitute

$$PV = nRT$$

$$\begin{aligned} n &= \frac{PV}{RT} = \frac{(1.50 \times 10^5 \text{ Pa})(1.00 \times 10^{-3} \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(293 \text{ K})} \\ &= 6.16 \times 10^{-2} \text{ mol} \end{aligned}$$

(b) Find the temperature after the gas expands to 2.00 L

$$\frac{P_f V_f}{P_i V_i} = \frac{nRT_f}{nRT_i}$$

Divide the ideal gas law for the final state by the ideal gas law for the initial state then, Cancel the number of moles n and the gas constant R , and solve for T_f

$$\frac{P_f V_f}{P_i V_i} = \frac{T_f}{T_i}$$

$$T_f = \frac{P_f V_f}{P_i V_i} T_i = \frac{(1.01 \times 10^5 \text{ Pa})(2.00 \text{ L})}{(1.50 \times 10^5 \text{ Pa})(1.00 \text{ L})} (293 \text{ K})$$

$$= 395 \text{ K}$$

As previously stated, the number of molecules contained in one mole of any gas is Avogadro's number, $N_A = 6.02 \times 10^{23}$ particles/mol, so

$$n = \frac{N}{N_A} \quad [1.10]$$

where n is the number of moles and N is the number of molecules in the gas. With Equation 1.10, we can rewrite the ideal gas law in terms of the total number of molecules as

$$PV = nRT = \frac{N}{N_A} RT$$

or $PV = Nk_B T$ [1.11]

where

$$k_B = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J/K} \quad [1.12]$$

is Boltzmann's constant. This reformulation of the ideal gas law will be used in the next section to relate the temperature of a gas to

the average kinetic energy of particles in the gas.

1.5 THE KINETIC THEORY OF GASES

In Section 1.4, we discussed the macroscopic properties of an ideal gas, including pressure, volume, number of moles, and temperature. In this section we consider the ideal gas model from the microscopic point of view.

We will show that the macroscopic properties can be understood on the basis of what is happening on the atomic scale. In addition, we reexamine the ideal gas law in terms of the behavior of the individual molecules that make up the gas.

Using the model of an ideal gas, we will describe the kinetic theory of gases. With this theory we can interpret the pressure and temperature of an ideal gas in terms of microscopic variables. The kinetic theory of gases model makes the following assumptions:

1. The number of molecules in the gas is large, and the average separation between them is large compared with their

dimensions. The fact that the number of molecules is large allows us to analyze their behavior statistically. The large separation between molecules means that the molecules occupy a negligible volume in the container. This assumption is consistent with the ideal gas model, in which we imagine the molecules to be pointlike.

2. The molecules obey Newton's laws of motion, but as a whole they move randomly. By "randomly" we mean that any molecule can move in any direction with equal probability, with a wide distribution of speeds.

3. The molecules interact only through short-range forces during elastic collisions. This assumption is consistent with the ideal gas model, in which the molecules exert no long-range forces on each other.

4. The molecules make elastic collisions with the walls.

5. All molecules in the gas are identical.

Although we often picture an ideal gas as consisting of single atoms, *molecular* gases exhibit ideal behavior at low pressures. On average, effects associated with molecular

structure have no effect on the motions considered, so we can apply the results of the following development to molecular gases as well as to monatomic gases.

Molecular Model for the Pressure of an Ideal Gas

As a first application of kinetic theory, we derive an expression for the pressure of an ideal gas in a container in terms of microscopic quantities. The pressure of the gas is the result of collisions between the gas molecules and the walls of the container. During these collisions, the gas molecules undergo a change of momentum as a result of the force exerted on them by the walls.

We now derive an expression for the pressure of an ideal gas consisting of N molecules in a container of volume V . In this section, we use m to represent the mass of one molecule. The container is a cube with edges of length d (Fig. 1.13).

Consider the collision of one molecule moving with a velocity $-v_x$ toward the left hand face of the box (Fig. 1.14).

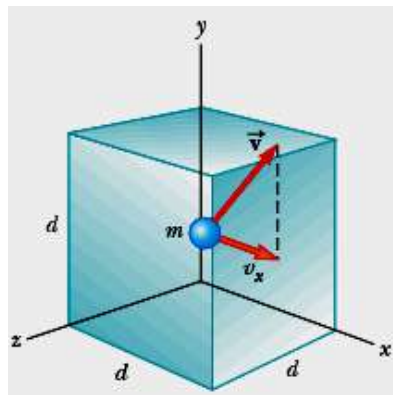


Figure 1.13 A cubical box with sides of length d containing an ideal gas.

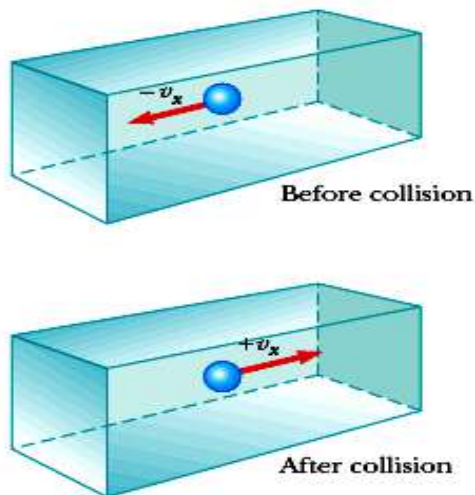


Figure 1.14 A molecule moving along the x - axis makes elastic collisions with the walls of the container. In colliding with a wall, the molecule's momentum is reversed, and the molecule exerts a force on the wall.

After colliding elastically with the wall, the molecule moves in the positive x -direction with a velocity $+v_x$. Because the momentum of the molecule is $-mv_x$ before the collision

and $+mv_x$ afterward, the change in its momentum is

$$\Delta p_x = mv_x - (-mv_x) = 2mv_x$$

If F_1 is the magnitude of the average force exerted by a molecule on the wall in the time Δt , then applying Newton's second law to the wall gives

$$F_1 = \frac{\Delta p_x}{\Delta t} = \frac{2mv_x}{\Delta t}$$

In order for the molecule to make two collisions with the same wall, it must travel a distance $2d$ along the x -direction in a time Δt . Therefore, the time interval between two collisions with the same wall is $\Delta t = 2d/v_x$, and the force imparted to the wall by a single molecule is

$$F_1 = \frac{2mv_x}{\Delta t} = \frac{2mv_x}{2d/v_x} = \frac{mv_x^2}{d}$$

The total force F exerted by all the molecules on the wall is found by adding the forces exerted by the individual molecules:

$$F = \frac{m}{d} (v_{1x}^2 + v_{2x}^2 + \dots)$$

In this equation, v_{1x} is the x -component of velocity of molecule 1, v_{2x} is the x -component of velocity of molecule 2, and so on.

The summation terminates when we reach N molecules because there are N molecules in the container.

Note that the average value of the square of the velocity in the x -direction for N molecules is

$$\overline{v_x^2} = \frac{v_{1x}^2 + v_{2x}^2 + \cdots + v_{Nx}^2}{N}$$

where $\overline{v_x^2}$ is the average value of v_x^2 . The total force on the wall can then be written

$$F = \frac{Nm}{d} \overline{v_x^2}$$

Now we focus on one molecule in the container traveling in some arbitrary direction with velocity and having components v_x , v_y , and v_z . In this case, we must express the total force on the wall in terms of the speed of the molecules rather than just a single component. The Pythagorean theorem relates the square of the speed to the square of these components according to the expression $v^2 = v_x^2 + v_y^2 + v_z^2$. Hence, the average value of v^2 for all the molecules in the container is related to the average values $\overline{v_x^2}$, $\overline{v_y^2}$, and $\overline{v_z^2}$ according to the expression $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$. Because the

motion is completely random, the average values $\overline{v_x^2}$, $\overline{v_y^2}$, and $\overline{v_z^2}$ are equal to each other. Using this fact and the earlier equation for v_x , we find that

$$\overline{v_x^2} = \frac{1}{3} \overline{v^2}$$

The total force on the wall, then, is

$$F = \frac{N}{3} \left(\frac{m \overline{v^2}}{d} \right)$$

This expression allows us to find the total pressure exerted on the wall by dividing the force by the area:

$$P = \frac{F}{A} = \frac{F}{d^2} = \frac{1}{3} \left(\frac{N}{d^3} m \overline{v^2} \right) = \frac{1}{3} \left(\frac{N}{V} \right) m \overline{v^2}$$

$$P = \frac{2}{3} \left(\frac{N}{V} \right) \left(\frac{1}{2} m \overline{v^2} \right) \quad [1.13]$$

Equation 1.13 says that the pressure is proportional to the number of molecules per unit volume and to the average translational kinetic energy of a molecule, $\frac{1}{2} m \overline{v^2}$. With this simplified model of an ideal gas, we have arrived at an important result that relates the large-scale quantity of pressure to an atomic quantity—the average value of the square of the molecular speed. This

relationship provides a key link between the atomic world and the large-scale world.

Equation 1.13 captures some familiar features of pressure. One way to increase the pressure inside a container is to increase the number of molecules per unit volume in the container.

You do this when you add air to a tire. The pressure in the tire can also be increased by increasing the average translational kinetic energy of the molecules in the tire. As we will see shortly, this can be accomplished by increasing the temperature of the gas inside the tire. That's why the pressure inside a tire increases as the tire warms up during long trips. The continuous flexing of the tires as they move along the road transfers energy to the air inside them, increasing the air's temperature, which in turn raises the pressure.

Molecular Interpretation of Temperature

Having related the pressure of a gas to the average kinetic energy of the gas molecules, we now relate temperature to a microscopic description of the gas. We can obtain some

insight into the meaning of temperature by multiplying Equation 1.13 by the volume:

$$PV = \frac{2}{3}N\left(\frac{1}{2}\overline{mv^2}\right)$$

Comparing this equation with the equation of state for an ideal gas in the form of Equation 1.11, $PV = Nk_B T$, we note that the left-hand sides of the two equations are identical. Equating the right-hand sides, we obtain

$$T = \frac{2}{3k_B} \left(\frac{1}{2}\overline{mv^2}\right) \quad [1. 14]$$

This means that the temperature of a gas is a direct measure of the average molecular kinetic energy of the gas. As the temperature of a gas increases, the molecules move with higher average kinetic energy. Rearranging Equation 1.14, we can relate the translational molecular kinetic energy to the temperature:

$$\frac{1}{2}\overline{mv^2} = \frac{3}{2}k_B T \quad [1. 15]$$

So the average translational kinetic energy per molecule is $\frac{3}{2}K_B T$. The total translational kinetic energy of N molecules of gas is simply N times the average energy per molecule,

$$KE_{\text{total}} = N\left(\frac{1}{2}\overline{mv^2}\right) = \frac{3}{2}Nk_B T = \frac{3}{2}nRT \quad [1. 16]$$

where we have used $k_B = R/N_A$ for Boltzmann's constant and $n = N/N_A$ for the number of moles of gas. From this result, we see that the total translational kinetic energy of a system of molecules is proportional to the absolute temperature of the system.

For a monatomic gas, translational kinetic energy is the only type of energy the molecules can have, so Equation 1.16 gives the internal energy U for a monatomic gas:

$$U = \frac{3}{2}nRT \quad (\text{monatomic gas}) \quad [1.17]$$

For diatomic and polyatomic molecules, additional possibilities for energy storage are available in the vibration and rotation of the molecule.

The square root of $\overline{v^2}$ is called the root-mean-square (rms) speed of the molecules. From Equation 1.15, we get, for the rms speed,

$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3RT}{M}} \quad [1.18]$$

where M is the molar mass in *kilograms per mole*, if R is given in SI units. Equation 1.18 shows that, at a given temperature, lighter molecules tend to move faster than heavier

molecules. For example, if gas in a vessel consists of a mixture of hydrogen and oxygen, the hydrogen (H_2) molecules, with a molar mass of $2.0 \times 10^{-3} \text{ kg/mol}$, move four times faster than the oxygen (O_2) molecules, with molar mass $32 \times 10^{-3} \text{ kg/mol}$. If we calculate the rms speed for hydrogen at room temperature ($\sim 300\text{K}$) we find

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31 \text{ J/mol} \cdot \text{K})(300 \text{ K})}{2.0 \times 10^{-3} \text{ kg/mol}}} = 1.9 \times 10^3 \text{ m/s}$$

This speed is about 17% of the escape speed for Earth. Because it is an average speed, a large number of molecules have much higher speeds and can therefore escape from Earth's atmosphere. This is why Earth's atmosphere doesn't currently contain hydrogen—it has all bled off into space.

EXAMPLE 1.9 A Cylinder of Helium

Goal Calculate the internal energy of a system and the average kinetic energy per molecule.

Problem A cylinder contains 2.00 mol of helium gas at 20.0°C. Assume that the helium behaves like an ideal gas.

(a) Find the total internal energy of the system. (b) What is the average kinetic energy per molecule? (c) How much energy would have to be added to the system to double the rms speed? The molar mass of helium is 4.00×10^{-3} kg/mol.

Solution

(a) Find the total internal energy of the system.

Substitute values into Equation 1.17 with $n = 2.00$ and

$T = 293$ K:

$$U = \frac{3}{2}(2.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(293 \text{ K}) = 7.30 \times 10^3 \text{ J}$$

(b) What is the average kinetic energy per molecule?

Substitute given values into Equation 1.15:

$$\begin{aligned} \frac{1}{2} m \overline{v^2} &= \frac{3}{2} k_B T = \frac{3}{2} (1.38 \times 10^{-23} \text{ J/K})(293 \text{ K}) \\ &= 6.07 \times 10^{-21} \text{ J} \end{aligned}$$

(c) How much energy must be added to double the rms speed?

From Equation 1.18, doubling the rms speed requires quadrupling T . Calculate the required change of internal energy, which is the energy that must be put into the system:

$$\Delta U = U_f - U_i = \frac{3}{2}nRT_f - \frac{3}{2}nRT_i = \frac{3}{2}nR(T_f - T_i)$$

$$\Delta U = \frac{3}{2}(2.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})((4.00 \times 293 \text{ K}) - 293 \text{ K}) = 2.19 \times 10^4 \text{ J}$$

CHAPTER 2

Energy in Thermal Processes

When two objects with different temperatures are placed in thermal contact, the temperature of the warmer object decreases while the temperature of the cooler object increases. With time, they reach a common equilibrium temperature somewhere in between their initial temperatures. During this process, we say that energy is transferred from the warmer object to the cooler one.

Until about 1850, the subjects of thermodynamics and mechanics were considered two distinct branches of science, and the principle of conservation of energy seemed to describe only certain kinds of mechanical systems. Experiments performed by the English physicist James Joule (1818–1889) and others showed that the decrease in mechanical energy (kinetic plus potential) of an isolated system was equal to the increase in internal energy of the system. Today, internal energy is treated as a form of

energy that can be transformed into mechanical energy and vice versa. Once the concept of energy was broadened to include internal energy, the law of conservation of energy emerged as a universal law of nature. This chapter focuses on some of the processes of energy transfer between a system and its surroundings.

2.1 HEAT AND INTERNAL ENERGY

A major distinction must be made between internal energy and heat. These terms are not interchangeable—heat involves a *transfer* of internal energy from one location to another. The following formal definitions will make the distinction precise.

Internal energy U is the energy associated with the microscopic components of a system—the atoms and molecules of the system. The internal energy includes kinetic and potential energy associated with the random translational, rotational, and vibrational motion of the particles that make up the system, and any potential energy bonding the particles together.

In Chapter 1 we showed that the internal energy of a monatomic ideal gas is associated with the translational motion of its atoms. In this special case, the internal energy is the total translational kinetic energy of the atoms; the higher the temperature of the gas, the greater the kinetic energy of the atoms and the greater the internal energy of the gas. For more complicated diatomic and polyatomic gases, internal energy includes other forms of molecular energy, such as rotational kinetic energy and the kinetic and potential energy associated with molecular vibrations. Internal energy is also associated with the intermolecular potential energy ("bond energy") between molecules in a liquid or solid.

Heat was considered to be one possible method of transferring energy between a system and its environment, and we provide a formal definition here:

Heat is the transfer of energy between a system and its environment due to a temperature difference between them.

The symbol Q is used to represent the amount of energy transferred by heat between a system and its environment. For brevity, we will often use the phrase “the energy Q transferred to a system . . .” rather than “the energy Q transferred by heat to a system . . .”

If a pan of water is heated on the burner of a stove, it's incorrect to say more heat is in the water. Heat is the *transfer* of thermal energy, just as work is the transfer of mechanical energy. When an object is pushed, it doesn't have more work; rather, it has more mechanical energy transferred *by* work. Similarly, the pan of water has more thermal energy transferred by heat.

Units of Heat

Early in the development of thermodynamics, before scientists realized the connection between thermodynamics and mechanics, heat was defined in terms of the temperature changes it produced in an object, and a separate unit of energy, the calorie, was used for heat. The calorie (cal) is defined as the energy necessary to raise the

temperature of 1 g of water from 14.5° to 15.5°C. (The “Calorie,” with a capital “C,” used in describing the energy content of foods, is actually a kilocalorie.)

Likewise, the unit of heat in the U.S. customary system, the British thermal unit (Btu), was defined as the energy required to raise the temperature of 1 lb of water from 63°F to 64°F.

In 1948, scientists agreed that because heat (like work) is a measure of the transfer of energy, its SI unit should be the joule. The calorie is now defined to be exactly 4.186 J:

$$1\text{Cal}=4.186\text{J} \quad [2.1]$$

This definition makes no reference to raising the temperature of water. The calorie is a general energy unit, introduced here for historical reasons, though we will make little use of it.

The definition in Equation 2.1 is known, from the historical background as *mechanical equivalent of heat*.

EXAMPLE 2.1 Working Off Breakfast

Goal Relate caloric energy to mechanical energy.

Problem A student eats a breakfast consisting of two bowls of cereal and milk, containing a total of 3.20×10^2 Calories of energy. He wishes to do an equivalent amount of work in the gymnasium by doing curls with a 25.0-kg barbell (Fig. 2.1). How many times must he raise the weight to expend that much energy? Assume that he raises it through a vertical displacement of 0.400 m each time, the distance from his lap to his upper chest.



Figure 2.1 (Example 2.1)

]

Solution

Convert his breakfast Calories, E , to joules:

$$\begin{aligned} E &= (3.20 \times 10^2 \text{ Cal}) \left(\frac{1.00 \times 10^3 \text{ cal}}{1.00 \text{ Cal}} \right) \left(\frac{4.186 \text{ J}}{\text{cal}} \right) \\ &= 1.34 \times 10^6 \text{ J} \end{aligned}$$

Use the work–energy theorem to find the work necessary to lift the barbell up to its maximum height.

$$W = \Delta KE + \Delta PE = (0 - 0) + (mgh - 0) = mgh$$

The student must expend the same amount of energy lowering the barbell, making $2mgh$ per repetition. Multiply this amount by n repetitions and set it equal to the food energy E :

$$n(2mgh) = E$$

Solve for n , substituting the food energy for E :

$$\begin{aligned} n &= \frac{E}{2mgh} = \frac{1.34 \times 10^6 \text{ J}}{2(25.0 \text{ kg})(9.80 \text{ m/s}^2)(0.400 \text{ m})} \\ &= 6.84 \times 10^3 \text{ times} \end{aligned}$$

2.2 SPECIFIC HEAT

The historical definition of the calorie is the amount of energy necessary to raise the temperature of one gram of a specific substance—water—by one degree. That amount is 4.186 J. Raising the temperature of one kilogram of water by 1° requires 4 186 J of energy. The amount of energy required to raise the temperature of one kilogram of an arbitrary substance by 1° varies with the substance. For example, the energy required to raise the temperature of one kilogram of copper by 1.0°C is 387 J. Every substance requires a unique amount of energy per unit mass to change the temperature of that substance by 1.0°C.

If a quantity of energy Q is transferred to a substance of mass m , changing its temperature by $\Delta T = T_f - T_i$, the specific heat c of the substance is defined by

$$C = \frac{Q}{m \Delta T} \quad [2.2]$$

SIunit: Joule per kilogram-degree Celsius
(J/kg .°C)

Table 2.1 lists specific heats for several substances. From the definition of the

calorie, the specific heat of water is 4 186 J/kg .°C.

From the definition of specific heat, we can express the energy Q needed to raise the temperature of a system of mass m by ΔT as

$$Q = mc \Delta T \quad [2.3]$$

TABLE 2-1

Specific Heats of Some Materials at Atmospheric Pressure		
Substance	J/kg · °C	cal/g · °C
Aluminum	900	0.215
Beryllium	1 820	0.436
Cadmium	230	0.055
Copper	387	0.0924
Germanium	322	0.077
Glass	837	0.200
Gold	129	0.0308
Ice	2 090	0.500
Iron	448	0.107
Lead	128	0.0305
Mercury	138	0.033
Silicon	703	0.168
Silver	234	0.056
Steam	2 010	0.480
Water	4 186	1.00

The energy required to raise the temperature of 0.500 kg of water by 3.00°C, for example, is

$$Q = (0.500 \text{ kg})(4 186 \text{ J/kg} \cdot ^\circ\text{C})(3.00^\circ\text{C}) = 6.28 \times 10^3 \text{ J}$$

Note that when the temperature increases, ΔT and Q are *positive*, corresponding to energy flowing *into* the system. When the temperature decreases, ΔT and Q are *negative*, and energy flows *out* of the system.

Table 2.1 shows that water has the highest specific heat relative to most other common substances. This high specific heat is responsible for the moderate temperatures found in regions near large bodies of water. As the temperature of a body of water decreases during winter, the water transfers energy to the air, which carries the energy landward when prevailing winds are toward the land. Off the western coast of the United States, the energy liberated by the Pacific Ocean is carried to the east, keeping coastal areas much warmer than they would otherwise be. Winters are generally colder in the eastern coastal states, because the prevailing winds tend to carry the energy away from land.

The fact that the specific heat of water is higher than the specific heat of sand is responsible for the pattern of airflow at a

beach. During the day, the Sun adds roughly equal amounts of energy to the beach and the water, but the lower specific heat of sand causes the beach to reach a higher temperature than the water. As a result, the air above the land reaches a higher temperature than the air above the water. The denser cold air pushes the less dense hot air upward (due to Archimedes's principle), resulting in a breeze from ocean to land during the day.

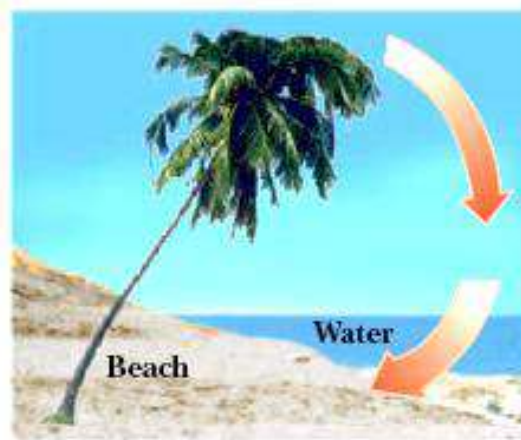


Figure 2.2 Circulation of air at the beach. On a hot day, the air above the sand warms faster than the air above the cooler water. The warmer air floats upward due to Archimedes's principle, resulting in the movement of cooler air toward the beach.

Because the hot air gradually cools as it rises, it subsequently sinks, setting up the circulation pattern shown in (Figure 2.2.)

A similar effect produces rising layers of air called *thermals* that can help eagles soar higher and hang gliders stay in flight longer. A thermal is created when a portion of the Earth reaches a higher temperature than neighboring regions. This often happens to plowed fields, which are warmed by the Sun to higher temperatures than nearby fields shaded by vegetation. The cooler, denser air over the vegetation-covered fields pushes the expanding air over the plowed field upwards, and a thermal is formed.

EXAMPLE 2.2 Stressing a Strut

Goal Use the energy transfer equation in the context of linear expansion and compressional stress.

Problem A steel strut near a ship's furnace is 2.00 m long, with a mass of 1.57 kg and cross-sectional area of $1.00 \times 10^{-4} \text{ m}^2$. During operation of the furnace, the strut absorbs thermal energy in a net amount of $2.50 \times 10^5 \text{ J}$.

(a) Find the change in temperature of the strut. (b) Find the increase in length of the strut. (c) If the strut is not allowed to expand because it's bolted at each end, find the compressional stress developed in the strut.

Solution

(a) Find the change in temperature.

Solve Equation 2.3 for the change in temperature and substitute:

$$Q = m_s c_s \Delta T \rightarrow \Delta T = \frac{Q}{m_s c_s}$$
$$\Delta T = \frac{(2.50 \times 10^5 \text{ J})}{(1.57 \text{ kg})(448 \text{ J/kg} \cdot ^\circ\text{C})} = 355^\circ\text{C}$$

(b) Find the change in length of the strut if it's allowed to expand.

Substitute into the linear expansion equation:

$$\Delta L = \alpha L_0 \Delta T = (11 \times 10^{-6} ^\circ\text{C}^{-1})(2.00 \text{ m})(355^\circ\text{C})$$
$$= 7.8 \times 10^{-3} \text{ m}$$

(c) Find the compressional stress in the strut if it is not allowed to expand.

Substitute into the compressional stress-strain equation:

$$\begin{aligned}\frac{F}{A} &= Y \frac{\Delta L}{L_0} = (2.00 \times 10^{11} \text{ Pa}) \frac{7.8 \times 10^{-3} \text{ m}}{2.01 \text{ m}} \\ &= 7.8 \times 10^8 \text{ Pa}\end{aligned}$$

2.3 CALORIMETRY

One technique for measuring the specific heat of a solid or liquid is to raise the temperature of the substance to some value, place it into a vessel containing cold water of known mass and temperature, and measure the temperature of the combination after equilibrium is reached. Define the system as the substance and the water. If the vessel is assumed to be a good insulator, so that energy doesn't leave the system, then we can assume the system is isolated. Vessels having this property are called calorimeters, and analysis performed using such vessels is called calorimetry.

The principle of conservation of energy for this isolated system requires that the net result of all energy transfers is zero. If one part of the system loses energy, another part has to gain the energy, because the system

is isolated and the energy has nowhere else to go. When a warm object is placed in the cooler water of a calorimeter, the warm object becomes cooler while the water becomes warmer. This principle can be written

$$Q_{\text{cold}} = -Q_{\text{hot}} \quad [2.4]$$

Q_{cold} is positive because energy is flowing into cooler objects, and Q_{hot} is negative because energy is leaving the hot object. The negative sign on the right-hand side of (Equation 2.4) ensures that the right-hand side is a positive number, consistent with the left-hand side. The equation is valid only when the system it describes is isolated. Calorimetry problems involve solving Equation 2.4 for an unknown quantity, usually either a specific heat or a temperature.

EXAMPLE 2.3 Finding a Specific Heat

Goal Solve a calorimetry problem involving only two substances.

Problem A 125-g block of an unknown substance with a temperature of 90.0°C is placed in a Styrofoam cup containing 0.326 kg of water at 20.0°C . The system reaches

an equilibrium temperature of 22.4°C. What is the specific heat, c_x , of the unknown substance if the heat capacity of the cup is neglected?

Solution

Let T be the final temperature, and let T_w and T_x be the initial temperatures of the water and block, respectively.

Apply Equations 2.3 and 2.4:

$$Q_{\text{cold}} = -Q_{\text{hot}}$$
$$m_w c_w (T - T_w) = -m_x c_x (T - T_x)$$

Solve for c_x and substitute numerical values:

$$c_x = \frac{m_w c_w (T - T_w)}{m_x (T_x - T)}$$
$$= \frac{(0.326 \text{ kg})(4190 \text{ J/kg} \cdot ^\circ\text{C})(22.4^\circ\text{C} - 20.0^\circ\text{C})}{(0.125 \text{ kg})(90.0^\circ\text{C} - 22.4^\circ\text{C})}$$
$$c_x = 388 \text{ J/kg} \cdot ^\circ\text{C}$$

As long as there are no more than two substances involved, Equation 2.4 can be used to solve elementary calorimetry problems. Sometimes, however, there may be three (or more) substances exchanging thermal energy, each at a different temperature. If the problem requires finding

the final temperature, it may not be clear whether the substance with the middle temperature gains or loses thermal energy. In such cases, Equation 2.4 can't be used reliably.

For example, suppose we want to calculate the final temperature of a system consisting initially of a glass beaker at 25°C, hot water at 40°C, and a block of aluminum at 37°C. We know that after the three are combined, the glass beaker warms up and the hot water cools, but we don't know for sure whether the aluminum block gains or loses energy because the final temperature is unknown.

Fortunately, we can still solve such a problem as long as it's set up correctly. With an unknown final temperature T_f , the expression $Q = mc(T_f - T_i)$ will be positive if $T_f > T_i$ and negative if $T_f < T_i$. Equation 2.4 can be written as

$$\sum Q_k = 0 \quad [2.5]$$

where Q_k is the energy change in the k th object. Equation 2.5 says that the sum of all the gains and losses of thermal energy must add up to zero, as required by the

conservation of energy for an isolated system. Each term in Equation 2.5 will have the correct sign automatically. Applying Equation 2.5 to the water, aluminum, and glass problem, we get

$$Q_w + Q_{al} + Q_g = 0$$

There's no need to decide in advance whether a substance in the system is gaining or losing energy. This equation is similar in style to the conservation of mechanical energy equation, where the gains and losses of kinetic and potential energies sum to zero for an isolated system: $\Delta K + \Delta PE = 0$. As will be seen, changes in thermal energy can be included on the left-hand side of this equation.

When more than two substances exchange thermal energy, it's easy to make errors substituting numbers, so it's a good idea to construct a table to organize and assemble all the data. This strategy is illustrated in the next example.

EXAMPLE 2.4 Calculate an Equilibrium Temperature

Goal Solve a calorimetry problem involving three substances at three different temperatures.

Problem Suppose 0.400 kg of water initially at 40.0°C is poured into a 0.300-kg glass beaker having a temperature of 25.0°C. A 0.500-kg block of aluminum at 37.0°C is placed in the water, and the system insulated. Calculate the final equilibrium temperature of the system.

Solution

Apply Equation 2.5 to the system:

$$Q_w + Q_{\text{al}} + Q_g = 0$$
$$m_w c_w (T - T_w) + m_{\text{al}} c_{\text{al}} (T - T_{\text{al}}) + m_g c_g (T - T_g) = 0 \quad (1)$$

Construct a data table:

Q (J)	m (kg)	c (J/kg °C)	T_f	T_i
Q_w	0.400	4 190	T	40.0°C
Q_{al}	0.500	9.00×10^2	T	37.0°C
Q_g	0.300	837	T	25.0°C

Using the table, substitute into Equation 1:

$$\begin{aligned}
 & (1.68 \times 10^3 \text{ J/}^\circ\text{C})(T - 40.0^\circ\text{C}) \\
 & \quad + (4.50 \times 10^2 \text{ J/}^\circ\text{C})(T - 37.0^\circ\text{C}) \\
 & \quad + (2.51 \times 10^2 \text{ J/}^\circ\text{C})(T - 25.0^\circ\text{C}) = 0 \\
 & (1.68 \times 10^3 \text{ J/}^\circ\text{C} + 4.50 \times 10^2 \text{ J/}^\circ\text{C} + 2.51 \times 10^2 \text{ J/}^\circ\text{C})T \\
 & \quad = 9.01 \times 10^4 \text{ J} \\
 & T = 37.9^\circ\text{C}
 \end{aligned}$$

2.4 LATENT HEAT AND PHASE CHANGE

A substance usually undergoes a change in temperature when energy is transferred between the substance and its environment. In some cases, however, the transfer of energy doesn't result in a change in temperature. This can occur when the physical characteristics of the substance change from one form to another, commonly referred to as a phase change. Some common phase changes are solid to liquid (melting), liquid to gas (boiling), and a change in the crystalline structure of a solid. Any such phase change involves a change in the internal energy, but *no change in the temperature*.

The energy Q needed to change the phase of a given pure substance is

$$Q = \pm mL \quad [2.6]$$

where L , called the latent heat of the substance, depends on the nature of the phase change as well as on the substance.

The unit of latent heat is the joule per kilogram (J/kg). The word *latent* means “lying hidden within a person or thing.” The positive sign in Equation 11.6 is chosen when energy is absorbed by a substance, as when ice is melting. The negative sign is chosen when energy is removed from a substance, as when steam condenses to water.

The latent heat of fusion L_f is used when a phase change occurs during melting or freezing, while the latent heat of vaporization L_v is used when a phase change occurs during boiling or condensing. For example, at atmospheric pressure the latent heat of fusion for water is 3.33×10^5 J/kg, and

TABLE 2.2

Latent Heats of Fusion and Vaporization						
Substance	Melting Point (°C)	Latent Heat of Fusion (J/kg) (cal/g)		Boiling Point (°C)	Latent Heat of Vaporization (J/kg) (cal/g)	
Helium	−269.65	5.23×10^3	(1.25)	−268.93	2.09×10^4	(4.99)
Nitrogen	−209.97	2.55×10^4	(6.09)	−195.81	2.01×10^5	(48.0)
Oxygen	−218.79	1.38×10^4	(3.30)	−182.97	2.13×10^5	(50.9)
Ethyl alcohol	−114	1.04×10^5	(24.9)	78	8.54×10^5	(204)
Water	0.00	3.33×10^5	(79.7)	100.00	2.26×10^6	(540)
Sulfur	119	3.81×10^4	(9.10)	444.60	3.26×10^5	(77.9)
Lead	327.3	2.45×10^4	(5.85)	1 750	8.70×10^5	(208)
Aluminum	660	3.97×10^5	(94.8)	2 450	1.14×10^7	(2 720)
Silver	960.80	8.82×10^4	(21.1)	2 193	2.33×10^6	(558)
Gold	1 063.00	6.44×10^4	(15.4)	2 660	1.58×10^6	(377)
Copper	1 083	1.34×10^5	(32.0)	1 187	5.06×10^6	(1 210)

the latent heat of vaporization for water is 2.26×10^6 J/kg. The latent heats of different substances vary considerably, as can be seen in Table 2.2.

Another process, sublimation, is the passage from the solid to the gaseous phase without going through a liquid phase. The fuming of dry ice (frozen carbon dioxide) illustrates this process, which has its own latent heat associated with it—the heat of sublimation.

EXAMPLE 2.5 Boiling Liquid Helium

Goal Apply the concept of latent heat of vaporization to liquid helium.

Problem Liquid helium has a very low boiling point, 4.2 K, as well as a low latent heat of vaporization, 2.09×10^4 J/kg. If energy is transferred to a container of liquid helium at the boiling point from an immersed electric

heater at a rate of 10.0W, how long does it take to boil away 2.00 kg of the liquid?

Solution

Find the energy needed to vaporize 2.00 kg of liquid

helium at its boiling point:

$$Q = mL_v = (2.00 \text{ kg})(2.09 \times 10^4 \text{ J/kg}) = 4.18 \times 10^4 \text{ J}$$

Divide this result by the power to find the time:

$$\Delta t = \frac{Q}{\mathcal{P}} = \frac{mL_v}{\mathcal{P}} = \frac{4.18 \times 10^4 \text{ J}}{10.0 \text{ W}}$$

$$\Delta t = 4.18 \times 10^3 \text{ s} = 69.7 \text{ min}$$

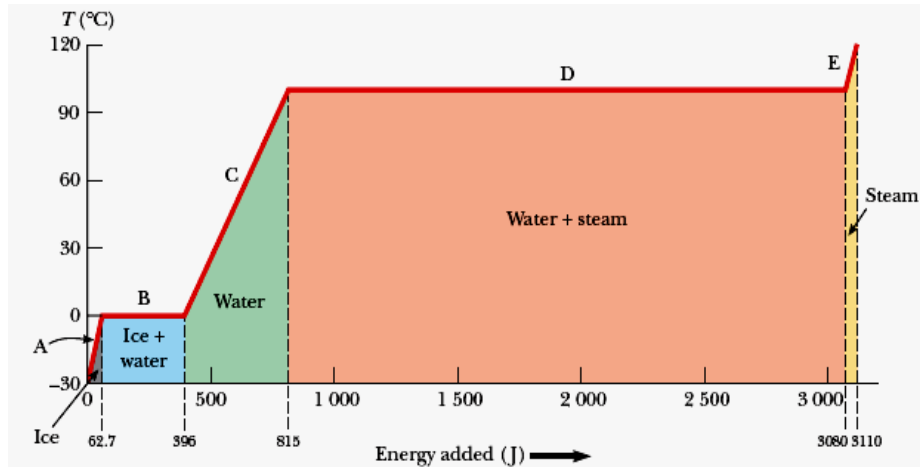


Figure 2.3 A plot of temperature versus energy added when 1.00 g of ice, initially at -30.0°C , is converted to steam at 120°C

To better understand the physics of phase changes, consider the addition of energy to a 1.00-g cube of ice at -30.0°C in a container held at constant pressure. Suppose this input of energy turns the ice to steam (water vapor) at 120.0°C . Figure 2.3 is a plot of the experimental measurement of temperature as energy is added to the system. We examine each portion of the curve separately.

Part A During this portion of the curve, the temperature of the ice changes from -30.0°C to 0.0°C . Because the specific heat of ice is $2\,090\text{ J/kg} \cdot ^{\circ}\text{C}$, we can calculate the amount of energy added from Equation 2.3:

$$Q = mc_{\text{ice}} \Delta T = (1.00 \times 10^{-3} \text{ kg})(2\,090 \text{ J/kg} \cdot ^{\circ}\text{C})(30.0^{\circ}\text{C}) = 62.7 \text{ J}$$

Part B When the ice reaches 0°C , the ice–water mixture remains at that temperature—even though energy is being added—until all the ice melts to become water at 0°C . According to Equation 11.6, the energy required to melt 1.00 g of ice at 0°C is

$$Q = mL_f = (1.00 \times 10^{-3} \text{ kg})(3.33 \times 10^5 \text{ J/kg}) = 333 \text{ J}$$

Part C Between 0°C and 100°C, no phase change occurs. The energy added to the water is used to increase its temperature, as in part A. The amount of energy necessary to increase the temperature from 0°C to 100°C is

$$Q = mc_{\text{water}} \Delta T = (1.00 \times 10^{-3} \text{ kg})(4.19 \times 10^3 \text{ J/kg} \cdot ^\circ\text{C})(1.00 \times 10^2 ^\circ\text{C})$$

$$Q = 4.19 \times 10^2 \text{ J}$$

Part D At 100°C, another phase change occurs as the water changes to steam at 100°C. As in Part B, the water–steam mixture remains at constant temperature, this time at 100°C—even though energy is being added—until all the liquid has been converted to steam. The energy required to convert 1.00 g of water at 100°C to steam at 100°C is

$$Q = mL_v = (1.00 \times 10^{-3} \text{ kg})(2.26 \times 10^6 \text{ J/kg}) = 2.26 \times 10^3 \text{ J}$$

Part E During this portion of the curve, as in parts A and C, no phase change occurs, so all the added energy goes into increasing the temperature of the steam. The energy that must be added to raise the temperature of the steam to 120.0°C is

$$Q = mc_{\text{steam}} \Delta T = (1.00 \times 10^{-3} \text{ kg})(2.01 \times 10^3 \text{ J/kg} \cdot ^\circ\text{C})(20.0^\circ\text{C}) = 40.2 \text{ J}$$

The total amount of energy that must be added to change 1.00 g of ice at -30.0°C to steam at 120.0°C is the sum of the results from all five parts of the curve— $3.11 \times 10^3 \text{ J}$. Conversely, to cool 1.00 g of steam at 120.0°C down to the point at which it becomes ice at -30.0°C , $3.11 \times 10^3 \text{ J}$ of energy must be removed.

Phase changes can be described in terms of rearrangements of molecules when energy is added to or removed from a substance. Consider first the liquid-to-gas phase change. The molecules in a liquid are close together, and the forces between them are stronger than the forces between the more widely separated molecules of a gas. Work must therefore be done on the liquid against these attractive molecular forces in order to separate the molecules. The latent heat of vaporization is the amount of energy that must be added to the one kilogram of liquid to accomplish this separation.

Similarly, at the melting point of a solid, the amplitude of vibration of the atoms about their equilibrium positions becomes great enough to allow the atoms to pass the barriers of adjacent atoms and move to their new positions. On average, these new positions are less symmetrical than the old ones and therefore have higher energy. The latent heat of fusion is equal to the work required at the molecular level to transform the mass from the ordered solid phase to the disordered liquid phase.

The average distance between atoms is much greater in the gas phase than in either the liquid or the solid phase. Each atom or molecule is removed from its neighbors, overcoming the attractive forces of nearby neighbors. Therefore, more work is required at the molecular level to vaporize a given mass of a substance than to melt it, so in general the latent heat of vaporization is much greater than the latent heat of fusion (Table 2.2).

EXAMPLE 2.6 Ice Water

Goal Solve a problem involving heat transfer and a phase change from solid to liquid.

Problem At a party, 6.00 kg of ice at -5.00°C is added to a cooler holding 30 liters of water at 20.0°C . What is the temperature of the water when it comes to equilibrium?

solution

Calculate the mass of liquid water:

$$\begin{aligned} m_{\text{water}} &= \rho_{\text{water}} V \\ &= (1.00 \times 10^3 \text{ kg/m}^3) (30.0 \text{ L}) \frac{1.00 \text{ m}^3}{1.00 \times 10^3 \text{ L}} \\ &= 30.0 \text{ kg} \end{aligned}$$

Write the equation of thermal equilibrium:

$$Q_{\text{ice}} + Q_{\text{melt}} + Q_{\text{ice-water}} + Q_{\text{water}} = 0$$

Construct a comprehensive table:

Q	m (kg)	c (J/kg $\cdot^{\circ}\text{C}$)	L (J/kg)	T_f ($^{\circ}\text{C}$)	T_i ($^{\circ}\text{C}$)	Expression
Q_{ice}	6.00	2 090		0	-5.00	$m_{\text{ice}} c_{\text{ice}} (T_f - T_i)$
Q_{melt}	6.00		3.33×10^5	0	0	$m_{\text{ice}} L_f$
$Q_{\text{ice-water}}$	6.00	4 190		T	0	$m_{\text{ice}} c_{\text{wat}} (T_f - T_i)$
Q_{water}	30.0	4 190		T	20.0	$m_{\text{wat}} c_{\text{wat}} (T_f - T_i)$

Substitute all quantities in the second through sixth columns into the last column and sum (which is the evaluation of Equation 1), and solve for T :

$$\begin{aligned}
 &6.27 \times 10^4 \text{ J} + 2.00 \times 10^6 \text{ J} \\
 &\quad + (2.51 \times 10^4 \text{ J/}^\circ\text{C})(T - 0^\circ\text{C}) \\
 &\quad + (1.26 \times 10^5 \text{ J/}^\circ\text{C})(T - 20.0^\circ\text{C}) = 0
 \end{aligned}$$

$$T = 3.03^\circ\text{C}$$

EXAMPLE 2.7 Partial Melting

Goal Understand how to handle an incomplete phase change.

Problem A 5.00-kg block of ice at 0°C is added to an insulated container partially filled with 10.0 kg of water at 15.0°C . (a) Find the final temperature, neglecting the heat capacity of the container. (b) Find the mass of the ice that was melted.

Solution

(a) Find the equilibrium temperature.

First, compute the amount of energy necessary to completely melt the ice:

$$\begin{aligned}
 Q_{\text{melt}} &= m_{\text{ice}}L_f = (5.00 \text{ kg})(3.33 \times 10^5 \text{ J/kg}) \\
 &= 1.67 \times 10^6 \text{ J}
 \end{aligned}$$

Next, calculate the maximum energy that can be lost by the initial mass of liquid water without freezing it:

$$\begin{aligned}
 Q_{\text{water}} &= m_{\text{water}} c \Delta T \\
 &= (10.0 \text{ kg})(4190 \text{ J/kg} \cdot ^\circ\text{C})(0^\circ\text{C} - 15.0^\circ\text{C}) \\
 &= -6.29 \times 10^5 \text{ J}
 \end{aligned}$$

This is less than half the energy necessary to melt all the ice, so the final state of the system is a mixture of water and ice at the freezing point:

$$T = 0^\circ\text{C}$$

(b) Compute the mass of ice melted.

Set the total available energy equal to the heat of fusion of m grams of ice, mL_f :

$$\begin{aligned}
 6.29 \times 10^5 \text{ J} &= mL_f = m(3.33 \times 10^5 \text{ J/kg}) \\
 m &= 1.89 \text{ kg}
 \end{aligned}$$

2.5 ENERGY TRANSFER

For some applications it's necessary to know the rate at which energy is transferred between a system and its surroundings and the mechanisms responsible for the transfer. This is particularly important in weatherproofing buildings or in medical applications, such as human survival time when exposed to the elements.

Earlier in this chapter we defined heat as a transfer of energy between a system and its surroundings due to a temperature difference between them. In this section, we take a closer look at heat as a means of energy transfer and consider the processes of thermal conduction, convection, and radiation.

Thermal Conduction

The energy transfer process most closely associated with a temperature difference is called thermal conduction or simply conduction. In this process, the transfer can be viewed on an atomic scale as an exchange of kinetic energy between microscopic particles—molecules, atoms, and electrons—with less energetic particles gaining energy as they collide with more energetic particles. An inexpensive pot may have a metal handle with no surrounding insulation. As the pot is warmed, the temperature of the metal handle increases, and the cook must hold it with a cloth potholder to avoid being burned.

The way the handle warms up can be understood by looking at what happens to the microscopic particles in the metal. Before the pot is placed on the stove, the particles are vibrating about their equilibrium positions. As the stove coil warms up, those particles in contact with it begin to vibrate with larger amplitudes. These particles collide with their neighbors and transfer some of their energy in the collisions. Metal atoms and electrons farther and farther from the flame gradually increase the amplitude of their vibrations, until eventually those in the metal near your hand are affected. This increased vibration represents an increase in temperature of the metal (and possibly a burned hand!).

Although the transfer of energy through a substance can be partly explained by atomic vibrations, the rate of conduction depends on the properties of the substance. For example, it's possible to hold a piece of asbestos in a flame indefinitely. This fact implies that very little energy is conducted through the asbestos. In general, metals are good thermal conductors because they contain

large numbers of electrons that are relatively free to move through the metal and can transport energy from one region to another. In a good conductor such as copper, conduction takes place via the vibration of atoms and the motion of free electrons. Materials such as asbestos, cork, paper, and fiberglass are poor thermal conductors. Gases are also poor thermal conductors because of the large distance between their molecules.

Conduction occurs only if there is a difference in temperature between two parts of the conducting medium. The temperature difference drives the flow of energy. Consider a slab of material of thickness Δx and cross-sectional area A with its opposite faces at different temperatures T_c and T_h , where $T_h > T_c$ (Fig. 2.4).

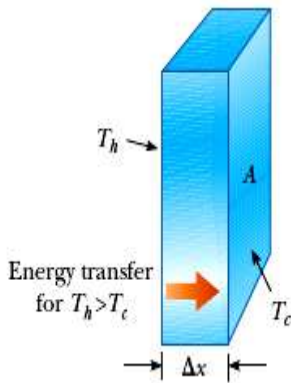
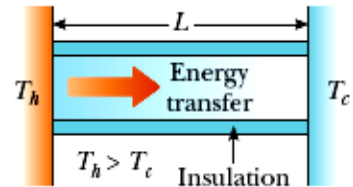


Figure 2.4



Energy transfer

Figure 2.5 Conduction of energy

through a conducting slab of cross-sectional area A and thickness L . The opposite faces are at different temperatures T_c and T_h .

through a uniform, insulated rod of length L . The opposite ends are in thermal contact with energy reservoirs at different temperatures.

The slab allows energy to transfer from the region of higher temperature to the region of lower temperature by thermal conduction. The rate of energy transfer, $\mathcal{P} = Q/\Delta t$, is proportional to the cross-sectional area of the slab and the temperature difference and is inversely proportional to the thickness of the slab:

$$\mathcal{P} = \frac{Q}{\Delta t} \propto A \frac{\Delta T}{\Delta x}$$

Note that $\mathcal{P}$ has units of watts when Q is in joules and Δt is in seconds.

Suppose a substance is in the shape of a long, uniform rod of length L , as in Figure 2.5. We assume the rod is insulated, so thermal energy can't escape by conduction from its surface except at the ends. One end is in thermal contact with an energy reservoir at temperature T_c and the other end is in thermal contact with a reservoir at temperature $T_h > T_c$. When a steady state is reached, the temperature at each point along the rod is constant in time. In this case, $\Delta T = T_h - T_c$ and

$\Delta x = L$, so The rate of energy transfer by conduction through the rod is given by

$$\frac{\Delta T}{\Delta x} = \frac{T_h - T_c}{L}$$

The rate of energy transfer by conduction through the rod is given by

$$\mathcal{P} = kA \frac{(T_h - T_c)}{L} \quad [2.7]$$

where k , a proportionality constant that depends on the material, is called the thermal conductivity. Substances that are good conductors have large thermal

conductivities, whereas good insulators have low thermal conductivities.

EXAMPLE 2.8 Energy Transfer through a Concrete Wall

Goal Apply the equation of heat conduction.

Problem Find the energy transferred in 1.00 h by conduction through a concrete wall 2.0 m high, 3.65 m long, and 0.20 m thick if one side of the wall is held at 20°C and the other side is at 5°C.

Solution

Multiply Equation 2.7 by Δt to find an expression for the total energy Q transferred through the wall:

$$Q = \mathcal{P} \Delta t = kA \left(\frac{T_h - T_c}{L} \right) \Delta t$$

Substitute the numerical values to obtain Q

$$\begin{aligned} Q &= (1.3 \text{ J/s} \cdot \text{m} \cdot ^\circ\text{C})(7.3 \text{ m}^2) \left(\frac{15^\circ\text{C}}{0.20 \text{ m}} \right) (3\,600 \text{ s}) \\ &= 2.6 \times 10^6 \text{ J} \end{aligned}$$

Remarks Early houses were insulated with thick masonry walls, which restrict energy loss by conduction because k is relatively low. The large thickness L also decreases

energy loss by conduction, as shown by Equation 2.7. There are much better insulating materials, however, and layering is also helpful. Despite the low thermal conductivity of masonry, the amount of energy lost is still rather large—enough to raise the temperature of 600 kg of water by more than 1°C. There are better insulating materials than masonry.

Home Insulation

To determine whether to add insulation to a ceiling or some other part of a building, the preceding discussion of conduction must be extended, for two reasons:

1. The insulating properties of materials used in buildings are usually expressed in engineering (U.S. customary) rather than SI units. Measurements stamped on a package of fiberglass insulating board will be in units such as British thermal units, feet, and degrees Fahrenheit.
2. In dealing with the insulation of a building, conduction through a compound slab must be considered, with each portion of the slab having a certain thickness and a specific thermal conductivity. A typical wall in a

house consists of an array of materials, such as wood paneling, drywall, insulation, sheathing, and wood siding.

The rate of energy transfer by conduction through a compound slab is

$$\frac{Q}{\Delta t} = \frac{A(T_h - T_c)}{\sum_i L_i / k_i} \quad [2.8]$$

where T_h and T_c are the temperatures of the *outer extremities* of the slab and the summation is over all portions of the slab. This formula can be derived algebraically, using the facts that the temperature at the interface between two insulating materials must be the same and that the rate of energy transfer through one insulator must be the same as through all the other insulators. If the slab consists of three different materials, the denominator is the sum of three terms. In engineering practice, the term L/k for a particular substance is referred to as the R Values for Some Common Building Materials. Equation 2.8 reduces to:

$$\frac{Q}{\Delta t} = \frac{A(T_h - T_c)}{\sum_i R_i} \quad [2.9]$$

The R values for a few common building materials are listed in Table 2.3. Note the unit of R and the fact that the R values are defined for specific thicknesses. Next to any vertical outside surface is a very thin, stagnant layer of air that must be considered when the total R value for a wall is computed. The thickness of this stagnant layer depends on the speed of the wind. As a result, energy loss by conduction from a house on a day when the wind is blowing is greater than energy loss on a day when the wind speed is zero. A representative R value for a stagnant air layer is given in Table 2.3

TABLE

2.3

R Values for Some Common Building Materials	
Material	R value (ft² · °F · h/Btu)
Hardwood siding (1.0 in. thick)	0.91
Wood shingles (lapped)	0.87
Brick (4.0 in. thick)	4.00
Concrete block (filled cores)	1.93
Styrofoam (1.0 in. thick)	5.0
Fiber glass batting (3.5 in. thick)	10.90
Fiber glass batting (6.0 in. thick)	18.80
Fiber glass board (1.0 in. thick)	4.35
Cellulose fiber (1.0 in. thick)	3.70
Flat glass (0.125 in. thick)	0.89
Insulating glass (0.25-in. space)	1.54
Vertical air space (3.5 in. thick)	1.01
Stagnant layer of air	0.17
Dry wall (0.50 in. thick)	0.45
Sheathing (0.50 in. thick)	1.32

Convection

When you warm your hands over an open flame, the air directly above the flame, being warmed, expands. As a result, the density of this air decreases and the air rises, warming your hands as it flows by. The transfer of energy by the movement of a substance is called convection. When the movement results from differences in density, as with air around a fire, it's referred to as *natural convection*. Airflow at a beach is an example

of natural convection, as is the mixing that occurs as surface water in a lake cools and sinks. When the substance is forced to move by a fan or pump, as in some hot air and hot water heating systems, the process is called *forced convection*.



Fig 2.6 Warming a hand by convection

Convection currents assist in the boiling of water. In a teakettle on a hot stovetop, the lower layers of water are warmed first. The warmed water has a lower density and rises to the top, while the denser, cool water at the surface sinks to the bottom of the kettle and is warmed.

The same process occurs when a radiator raises the temperature of a room. The hot radiator warms the air in the lower regions of the room. The warm air expands and, because of its lower density, rises to the

ceiling. The denser cooler air from above sinks, setting up the continuous air current pattern shown in Figure 2.7.



Figure 2 .7 Convection currents are set up in a room warmed by a radiator.

An automobile engine is maintained at a safe operating temperature by a combination of conduction and forced convection. Water (actually, a mixture of water and antifreeze) circulates in the interior of the engine. As the metal of the engine block increases in temperature, energy passes from the hot metal to the cooler water by thermal conduction. The water pump forces water out of the engine and into the radiator, carrying energy along with it (by forced convection). In the radiator, the hot water passes through metal pipes that are in contact with the

cooler outside air, and energy passes into the air by conduction. The cooled water is then returned to the engine by the water pump to absorb more energy. The process of air being pulled past the radiator by the fan is also forced convection.

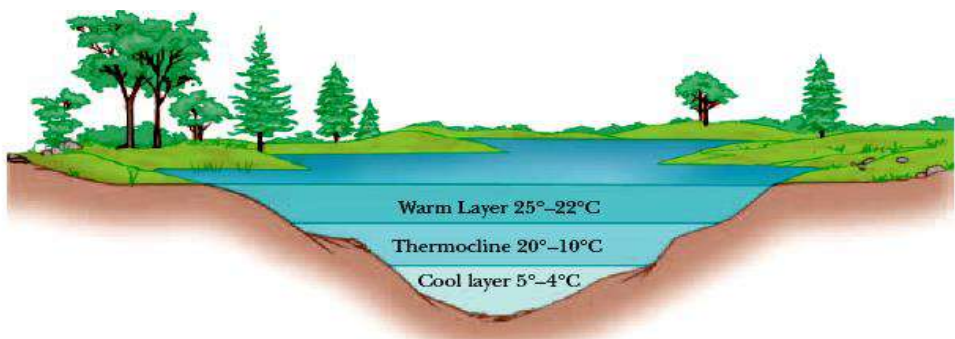
The algal blooms often seen in temperate lakes and ponds during the spring or fall are caused by convection currents in the water. To understand this process, consider Figure 2.8. During the summer, bodies of water develop temperature gradients, with an upper, warm layer of water separated from a lower, cold layer by a buffer zone called a thermocline. In the spring or fall, temperature changes in the water break down this thermocline, setting up convection currents that mix the water. The mixing process transports nutrients from the bottom to the surface. The nutrient-rich water forming at the surface can cause a rapid, temporary increase in the algae population.

Body Temperature

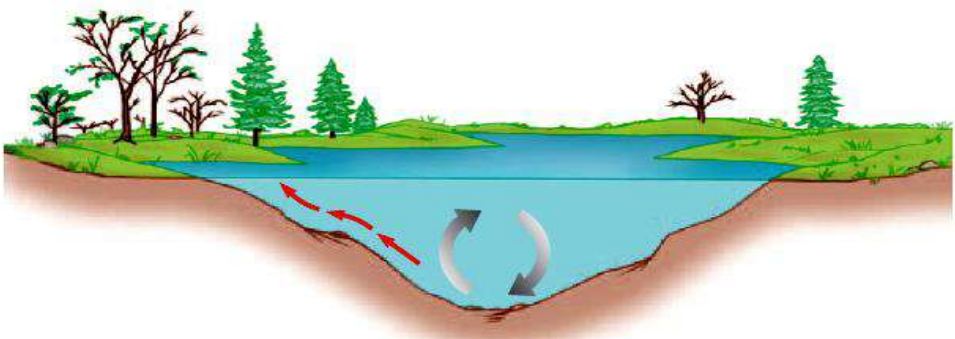
The body temperature of mammals ranges from about 35°C to 38°C, while that of birds

ranges from about 40°C to 43°C. How can these narrow ranges of body temperature be maintained in cold weather?

Explanation A natural method of maintaining body temperature is via layers of fat beneath the skin. Fat protects against both conduction and convection because of its low thermal conductivity and because there are few blood vessels in fat to carry blood to the surface, where energy losses by



(a) Summer layering of water



(b) Fall and spring upwelling

Figure 2.8 (a) During the summer, a warm upper layer of water is separated from a cooler lower layer by a thermocline. (b) Convection currents during the spring or fall mix the water and can cause algal blooms

convection can occur. Birds ruffle their feathers in cold weather in order to trap a layer of air with a low thermal conductivity between the feathers and the skin. Bristling the fur produces the same effect in fur-bearing animals. Humans keep warm with wool sweaters and down jackets that trap the warmer air in regions close to their bodies, reducing energy loss by convection and conduction

Radiation

Another process of transferring energy is through radiation. your hands can be warmed at an open flame through radiation. Because your hands aren't in physical contact with the flame and the conductivity of air is very low, conduction can't account for the energy transfer. Nor can convection be responsible for any transfer of energy, because your hands aren't above the flame in the path of convection currents. The warmth felt in your

hands, therefore, must come from the transfer of energy by radiation.

All objects radiate energy continuously in the form of electromagnetic waves due to thermal vibrations of their molecules. These vibrations create the orange glow of an electric stove burner, an electric space heater, and the coils of a toaster.

The rate at which an object radiates energy is proportional to the fourth power of its absolute temperature. This is known as Stefan's law, expressed in equation form as

$$\mathcal{P} = \sigma A e T^4 \quad [2.10]$$

where $\mathcal{P}$ is the power in watts (or joules per second) radiated by the object, σ is the Stefan–Boltzmann constant, equal to $5.6696 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$, A is the surface area of the object in square meters, e is a constant called the emissivity of the object, and T is the object's Kelvin temperature. The value of e can vary between zero and one, depending on the properties of the object's surface. Approximately 1340 J of electromagnetic radiation from the Sun passes through each square meter at the top of the Earth's atmosphere every second.

This radiation is primarily visible light, accompanied by significant amounts of infrared and ultraviolet. Some of this energy is reflected back into space, and some is absorbed by the atmosphere, but enough arrives at the surface of the Earth each day to supply all our energy needs hundreds of times over—if it could be captured and used efficiently. The growth in the number of solar houses in the United States is one example of an attempt to make use of this abundant energy. Radiant energy from the Sun affects our day-to-day existence in a number of ways, influencing Earth's average temperature, ocean currents, agriculture, and rain patterns. It can also affect behavior. As another example of the effects of energy transfer by radiation, consider what happens to the atmospheric temperature at night. If there is a cloud cover above

Earth, the water vapor in the clouds absorbs part of the infrared radiation emitted by Earth and re-emits it back to the surface. Consequently, the temperature at the surface remains at moderate levels. In the absence of cloud cover, there is nothing to

prevent the radiation from escaping into space, so the temperature drops more on a clear night than when it's cloudy.

As an object radiates energy at a rate given by Equation 2.10, it also absorbs radiation. If it didn't, the object would eventually radiate all its energy and its temperature would reach absolute zero. The energy an object absorbs comes from its environment, which consists of other bodies that radiate energy. If an object is at a temperature T , and its surroundings are at a temperature T_0 , the net energy gained or lost each second by the object as a result of radiation is

$$\mathcal{P}_{\text{net}} = \sigma A e (T^4 - T_0^4) \quad [2.11]$$

When an object is in equilibrium with its surroundings, it radiates and absorbs energy at the same rate, so its temperature remains constant. When an object is hotter than its surroundings, it radiates more energy than it absorbs and so cools.

An ideal absorber is an object that absorbs all the light radiation incident on it, including invisible infrared and ultraviolet light. Such an object is called a black body because a room-temperature black body would look

black. Since a black body doesn't reflect radiation at any wavelength, any light coming from it is due to atomic and molecular vibrations alone. A perfect black body has emissivity $e = 1$.

An ideal absorber is also an ideal radiator of energy. The Sun, for example, is nearly a perfect black body. This statement may seem contradictory, because the Sun is bright, not dark; however, the light that comes from the Sun is emitted, not reflected. Black bodies are perfect absorbers that look black at room temperature because they don't reflect any light. All black bodies, except those at absolute zero, emit light that has a characteristic spectrum. In contrast to black bodies, an object for which $e = 0$ absorbs none of the energy incident on it, reflecting it all. Such a body is an ideal reflector.

White clothing is more comfortable to wear in the summer than black clothing.

Black fabric acts as a good absorber of incoming sunlight and as a good emitter of this absorbed energy. About half of the emitted energy, however, travels toward the body, causing the person wearing the

garment to feel uncomfortably warm. White or light-colored clothing reflects away much of the incoming energy.

The amount of energy radiated by an object can be measured with temperature- sensitive recording equipment via a technique called thermography. An image of the pattern formed by varying radiation levels, called a thermogram, is brightest in the warmest areas. Figure 2.9 reproduces a thermogram of a house.

More energy escapes in the lighter regions, such as the door and windows. The owners of this house could conserve energy and reduce their heating costs by adding insulation to the attic area and by installing thermal draperies over the windows. Thermograms have also been used to image injured or diseased tissue in medicine, since such areas are often at a different temperature than surrounding healthy tissue, though many radiologists consider thermograms inadequate as a diagnostic tool.



Figure 2.9 This thermogram of a house, made during cold weather, shows colors ranging from white and yellow (areas of greatest energy loss) to blue and purple (areas of least energy loss).

APPLICATION

Thermal Radiation and Night Vision

How can thermal radiation be used to see objects in near total darkness?

Explanation There are two methods of night vision, one enhancing a combination of very faint visible light and infrared light, and another using infrared light only. The latter is valuable for creating images in absolute darkness. Because all objects above absolute zero emit thermal radiation due to the vibrations of their atoms, the infrared (invisible) light can be focused by a special lens and scanned by an array of infrared

detector elements. These elements create a thermogram. The information from thousands of separate points in the field of view is converted to electrical impulses and translated by a microchip into a form suitable for display. Different temperature areas are assigned different colors, which can then be easily discerned on the display.

EXAMPLE 2.12 Polar Bear Club

Goal Apply Stefan's law.

Problem A member of the Polar Bear Club, dressed only in bathing trunks of negligible size, prepares to plunge into the Baltic Sea from the beach in St. Petersburg, Russia. The air is calm, with a temperature of 5°C . If the swimmer's surface body temperature is 25°C , compute the net rate of energy loss from his skin due to radiation. How much energy is lost in 10.0 min? Assume his emissivity is 0.900, and his surface area is 1.50 m^2 .

Solution

Convert temperatures from Celsius to Kelvin:

$$T_{5^{\circ}\text{C}} = T_{\text{C}} + 273 = 5 + 273 = 278 \text{ K}$$

$$T_{25^{\circ}\text{C}} = T_{\text{C}} + 273 = 25 + 273 = 298 \text{ K}$$

Compute the net rate of energy loss, using Equation 11.11:

$$\begin{aligned}\mathcal{P}_{\text{net}} &= \sigma A_e (T^4 - T_0^4) \\ &= (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4) (1.50 \text{ m}^2) \\ &\quad \times (0.90) [(298 \text{ K})^4 - (278 \text{ K})^4] \\ \mathcal{P}_{\text{net}} &= 146 \text{ W}\end{aligned}$$

Multiply the preceding result by the time, 10 minutes, to get the energy lost in that time due to radiation:

$$Q = \mathcal{P}_{\text{net}} \times \Delta t = (146) (6.00 \times 10^2 \text{ s}) = 8.76 \times 10^4 \text{ J}$$

The Dewar Flask

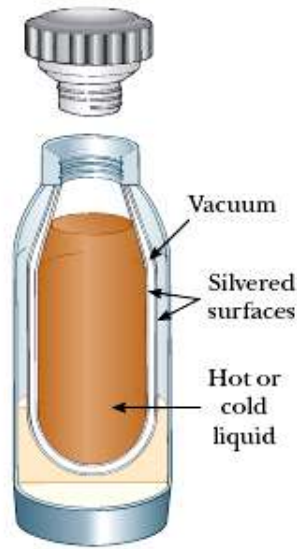


Figure 2.10 A cross-sectional view of a Thermos bottle designed to store hot or cold liquids.

The Thermos bottle, also called a Dewar flask (after its inventor), is designed to minimize energy transfer by conduction, convection, and radiation. The thermos can store either cold or hot liquids for long periods. The standard vessel (Fig. 2.10) is a double-walled Pyrex glass with silvered walls. The space between the walls is evacuated to minimize energy transfer by conduction and convection. The silvered surface minimizes energy transfer by radiation because silver is a very good reflector and has very low emissivity. A further reduction in energy loss is achieved by reducing the size of the neck.

Dewar flasks are commonly used to store liquid nitrogen (boiling point 77 K) and liquid oxygen (boiling point 90 K).

To confine liquid helium (boiling point 4.2 K), which has a very low heat of vaporization, it's often necessary to use a double Dewar system in which the Dewar flask containing the liquid is surrounded by a second Dewar flask. The space between the two flasks is filled with liquid nitrogen.

Some of the principles of the Thermos bottle are used in the protection of sensitive electronic instruments in orbiting space satellites. In half of its orbit around the Earth a satellite is exposed to intense radiation from the Sun, and in the other half it lies in the Earth's cold shadow. Without protection, its interior would be subjected to tremendous extremes of temperature. The interior of the satellite is wrapped with blankets of highly reflective aluminum foil. The foil's shiny surface reflects away much of the Sun's radiation while the satellite is in the unshaded part of the orbit and helps retain interior energy while the satellite is in the Earth's shadow.

2.6 GLOBAL WARMING AND GREENHOUSE GASES

Many of the principles of energy transfer, and opposition to it, can be understood by studying the operation of a glass greenhouse. During the day, sunlight passes into the greenhouse and is absorbed by the walls, soil, plants, and so on. This absorbed visible light is subsequently reradiated as infrared radiation, causing the temperature of the interior to rise.

In addition, convection currents are inhibited in a greenhouse. As a result, warmed air can't rapidly pass over the surfaces of the greenhouse that are exposed to the outside air and thereby cause an energy loss by conduction through those surfaces. Most experts now consider this restriction to be a more important warming effect than the trapping of infrared radiation. In fact, experiments have shown that when the glass over a greenhouse is replaced by a special glass known to transmit infrared light, the temperature inside is lowered only

slightly. On the basis of this evidence, the primary mechanism that raises the temperature of a greenhouse is not the trapping of infrared radiation, but the inhibition of airflow that occurs under any roof (in an attic, for example).

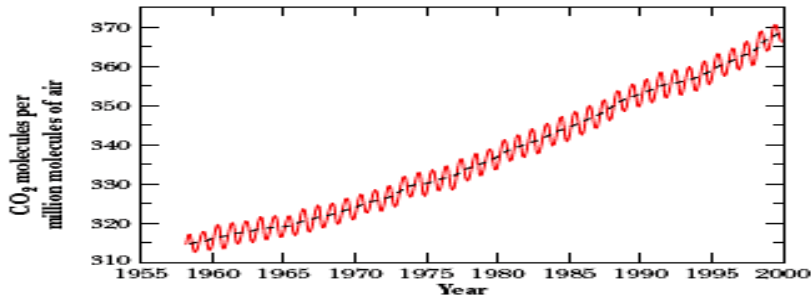


Figure 2.11 The concentration of atmospheric carbon dioxide in parts per million (ppm) of dry air as a function of time during the latter part of the 20th century. These data were recorded at Mauna Loa Observatory in Hawaii. The yearly variations (red curve) coincide with growing seasons, because vegetation absorbs carbon dioxide from the air. The steady increase (black curve) is of concern to scientists.

A phenomenon commonly known as the greenhouse effect can also play a major role in determining the Earth's temperature. First, note that the Earth's atmosphere is a good transmitter (and hence a poor absorber) of visible radiation and a good absorber of infrared radiation. The visible light that reaches the Earth's surface is absorbed and reradiated as infrared light, which in turn is

absorbed (trapped) by the Earth's atmosphere. An extreme case is the warmest planet, Venus, which has a carbon dioxide (CO₂) atmosphere and temperatures approaching 850°F.

As fossil fuels (coal, oil, and natural gas) are burned, large amounts of carbon dioxide are released into the atmosphere, causing it to retain more energy. This is of great concern to scientists and governments throughout the world. Many scientists are convinced that the 10% increase in the amount of atmospheric carbon dioxide in the past 30 years could lead to drastic changes in world climate.

The increase in concentration of atmospheric carbon dioxide in the latter part of the 20th century is shown in Figure 2.11. According to one estimate, doubling the carbon dioxide content in the atmosphere will cause temperatures to increase by 2°C. In temperate regions, such as Europe and the United States, a 2°C temperature rise would save billions of dollars per year in fuel costs. Unfortunately, it would also melt a large amount of ice from the polar ice caps, which

could cause flooding and destroy many coastal areas. A 2°C rise would also increase the frequency of droughts, and consequently decrease already low crop yields in tropical and subtropical countries. Even slightly higher average temperatures might make it impossible for certain plants and animals to survive in their customary ranges.

At present, about 3.5×10^{11} tons of CO₂ are released into the atmosphere each year. Most of this gas results from human activities such as the burning of fossil fuels, the cutting of forests, and manufacturing processes. Another greenhouse gas is methane (CH₄), which is released in the digestive process of cows and other ruminants. This gas originates from that part of the animal's stomach called the *rumen*, where cellulose is digested. Termites are also major producers of this gas. Finally, greenhouse gases such as nitrous oxide (N₂O) and sulfur dioxide (SO₂) are increasing due to automobile and industrial pollution.

Whether the increasing greenhouse gases are responsible or not, there is convincing evidence that global warming is underway.

The evidence comes from the melting of ice in Antarctica and the retreat of glaciers at widely scattered sites throughout the world (see Fig. 2.12). For example, satellite images of Antarctica show James Ross Island completely surrounded by water for the first time since maps were made, about 100 years ago. Previously, the island was connected to the mainland by an ice bridge. In addition, at various places across the continent, ice shelves are retreating, some at a rapid rate.



Figure 2.12 Glacier melting in Alaska. (a) Hiker on a ridge above the Muir Glacier in Glacier Bay National Park, August 26, 1978. (b) Hiker on the same ridge, June 27, 1993.

Perhaps at no place in the world are glaciers monitored with greater interest than in Switzerland. There, it is found that the Alps have lost about 50% of their glacial ice compared to 130 years ago. The retreat of glaciers on high-altitude peaks in the tropics is even more severe than in Switzerland. The Lewis glacier on Mount Kenya and the snows of Kilimanjaro are two examples. However, in certain regions of the planet where glaciers are near large bodies of water and are fed by large and frequent snows, glaciers continue to advance, so the overall picture of a catastrophic global-warming scenario may be premature. In about 50 years, however, the amount of carbon dioxide in the atmosphere is expected to be about twice what it was in the preindustrial era. Because of the possible catastrophic consequences, most scientists voice the concern that reductions in greenhouse gas emissions need to be made now.



**Zagazig University
Faculty of Science
Physics Department**



Physics of **SOUND**

**For
First Year Students
Semester 1**

Chapter 1

Oscillatory Motion

Periodic motion is motion of an object that regularly repeats—the object returns to a given position after a fixed time interval. With a little thought, we can identify several types of periodic motion in everyday life. Your car returns to the driveway each after-noon. You return to the dinner table each night to eat. A bumped chandelier swings back and forth, returning to the same position at a regular rate. The Earth returns to the same position in its orbit around the Sun each year, resulting in the variation among the four seasons. The Moon returns to the same relationship with the Earth and the Sun, resulting in a full Moon approximately once a month. In addition to these everyday examples, numerous other systems exhibit periodic motion. For example, the molecules in a solid oscillate about their equilibrium positions; electromagnetic waves, such as light waves, radar, and radio waves, are characterized by oscillating electric and magnetic field vectors; and in alternating-current electrical circuits, voltage, current, and electric charge vary periodically with time.

A special kind of periodic motion occurs in mechanical systems when the force acting on an object is proportional to the position of the object relative to some equilibrium position. If this force is always directed toward the equilibrium position, the motion is called simple harmonic motion, which is the primary focus of this chapter.

1.1 Motion of an Object Attached to a Spring

As a model for simple harmonic motion, consider a block of mass m attached to the end of a spring, with the block free to move on a horizontal,

frictionless surface (Fig.1.1). When the spring is neither stretched nor compressed, the block is at the position called the equilibrium position of the system, which we identify as $x = 0$. We know from experience that such a system oscillates back and forth if disturbed from its equilibrium position.

We can understand the motion in Figure 1.1 qualitatively by first recalling that when the block is displaced to a position x , the spring exerts on the block a force that

is proportional to the position and given by Hooke's law

$$F_s = -kx \quad (1.1)$$

We call this a restoring force because it is always directed toward the equilibrium position and therefore opposite the displacement from equilibrium. That is, when the block is displaced to the right of $x = 0$ in Figure 1.1, then the position is positive and the restoring force is directed to the left. When the block is displaced to the left of $x = 0$, then the position is negative and the restoring force is directed to the right.

Applying Newton's second law $\Sigma F_x = ma_x$ to the motion of the block, with Equation 1.1 providing the net force in the x direction, we obtain

$$\begin{aligned} -kx &= ma_x \\ a_x &= -\frac{k}{m}x \end{aligned} \quad (1.2)$$

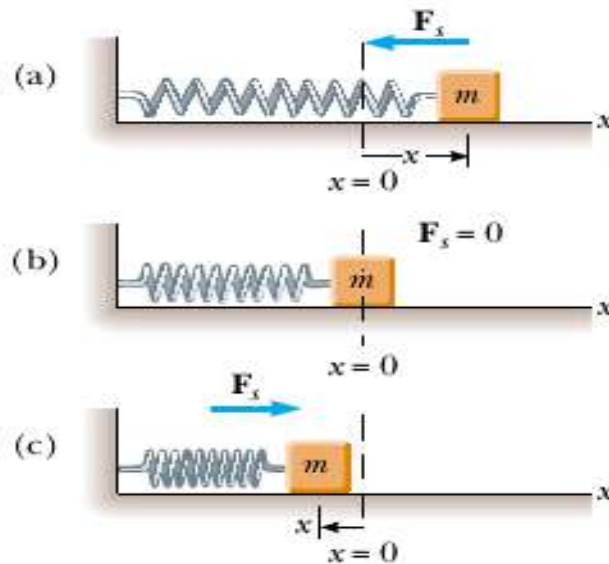


Figure 1.1 A block attached to a spring moving on a frictionless surface. (a) When the block is displaced to the right of equilibrium ($x > 0$), the force exerted by the spring acts to the left. (b) When the block is at its equilibrium position ($x = 0$), the force exerted by the spring is zero. (c) When the block is displaced to the left of equilibrium ($x < 0$), the force exerted by the spring acts to the right.

That is, the acceleration is proportional to the position of the block, and its direction is opposite the direction of the displacement from equilibrium. Systems that behave in this way are said to exhibit simple harmonic motion. An object moves with simple harmonic motion whenever its acceleration is proportional to its position and is oppositely directed to the displacement from equilibrium.

If the block in Figure 1.1 is displaced to a position $x = A$ and released from rest, its initial acceleration is $-kA/m$. When the block passes through the equilibrium position $x = 0$, its acceleration is zero. At this instant, its speed is a maximum because the acceleration changes sign. The block then continues to travel to the left of equilibrium with a positive acceleration and finally reaches $x = -A$, at which time its acceleration is $+kA/m$ and its speed

is again zero. The block completes a full cycle of its motion by returning to the original position, again passing through $x=0$ with maximum speed. Thus, we see that the block oscillates between the turning points $x=\pm A$. In the absence of friction, because the force exerted by the spring is conservative, this idealized motion will continue forever. Real systems are generally subject to friction, so they do not oscillate forever. We explore the details of the situation with friction in Section 1.6.

As Pitfall Prevention 1.1 points out, the principles that we develop in this chapter are also valid for an object hanging from a vertical spring, as long as we recognize that the weight of the object will stretch the spring to a new equilibrium position $x=0$. To prove this statement, let x_s represent the total extension of the spring from its equilibrium position without the hanging object. Then, $x_s = -(mg/k) + x$, where $-(mg/k)$ is the extension of the spring due to the weight of the hanging object and x is the instantaneous extension of the spring due to the simple harmonic motion. The magnitude of the net force on the object is then $F_s - F_g = -k(-(mg/k) + x) - mg = -kx$. The net force on the object is the same as that on a block connected to a horizontal spring as in Equation 1.1, so the same simple harmonic motion results.

1.2 Mathematical Representation of Simple Harmonic Motion

Let us now develop a mathematical representation of the motion we described in the preceding section. We model the block as a particle subject to the force in Equation 1.1. We will generally choose x as the axis along which the oscillation occurs; hence, we will drop the subscript- x notation in

this discussion. Recall that, by definition, $a = dv/dt = d^2x/dt^2$, and so we can express Equation 1.2 as

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \quad (1.3)$$

If we denote the ratio k/m with the symbol ω^2 (we choose ω^2 rather than 'in order to make the solution that we develop below simpler in form), then

$$\omega^2 = \frac{k}{m} \quad (1.4)$$

and Equation 1.3 can be written in the form

$$\frac{d^2x}{dt^2} = -\omega^2x \quad (1.5)$$

What we now require is a mathematical solution to Equation 1.5—that is, a function $x(t)$ that satisfies this second-order differential equation. This is a mathematical representation of the position of the particle as a function of time. We seek a function $x(t)$ whose second derivative is the same as the original function with a negative sign and multiplied by ω^2 . The trigonometric functions sine and cosine exhibit this behavior, so we can build a solution around one or both of these. The following cosine function is a solution to the differential equation:

$$x(t) = A \cos(\omega t + \phi) \quad (1.6)$$

where A , ω , and ϕ are constants. To see explicitly that this equation satisfies Equation 1.5, note that

$$\frac{dx}{dt} = A \frac{d}{dt} \cos(\omega t + \phi) = -\omega A \sin(\omega t + \phi) \quad (1.7)$$

$$\frac{d^2x}{dt^2} = \omega A \frac{d}{dt} \sin(\omega t + \phi) = -\omega^2 A \cos(\omega t + \phi) \quad (1.8)$$

Comparing Equations 1.6 and 1.8, we see that $d^2x/dt^2 = -\omega^2x$ and Equation 1.5 is satisfied.

The parameters A , ω , and ϕ are constants of the motion. In order to give physical significance to these constants, it is convenient to form a graphical representation of the motion by plotting x as a function of t , as in Figure 1.2a. First, note that A , called the amplitude of the motion, is simply the maximum value of the position of the particle in either the positive or negative x direction. The constant ω is called the angular frequency, and has units of rad/s. It is a measure of how rapidly the oscillations are occurring—the more oscillations per unit time, the higher is the value of ω . From Equation 1.4, the angular frequency is

$$\omega = \sqrt{\frac{k}{m}} \quad (1.9)$$

The constant angle ϕ (is called the phase constant (or initial phase angle) and, along with the amplitude A , is determined uniquely by the position and velocity of the particle at $t = 0$. If the particle is at its maximum position $x = A$ at $t = 0$, the phase constant is $\phi = 0$ and the graphical representation of the motion is shown in Figure 1.2b. The quantity $(\omega t + \phi)$ is called the phase of the motion. Note that the function $x(t)$ is periodic and its value is the same each time ωt increases by 2π radians.

Equations 1.1, 1.5, and 1.6 form the basis of the mathematical representation of simple harmonic motion. If we are analyzing a situation and find that the force on a particle is of the mathematical form of Equation 1.1, we know that the motion will be that of a simple harmonic oscillator and

that the position of the particle is described by Equation 1.6. If we analyze a system and find that it is described by a differential equation of the form of Equation 1.5, the motion will be that of a simple harmonic oscillator. If we analyze a situation and find that the position of a particle is described by Equation 1.6, we know the particle is undergoing simple harmonic motion.

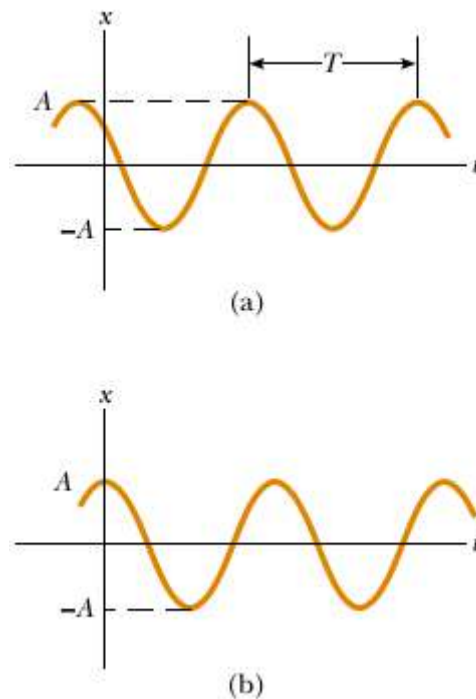


Figure 1.2(a) An x -vs.- t graph for an object undergoing simple harmonic motion. The amplitude of the motion is A , the period is T , and the phase constant is ϕ . (b) The x -vs.- t graph in the special case in which $x=A$ at $t = 0$ and hence $\phi = 0$.

An experimental arrangement that exhibits simple harmonic motion is illustrated in Figure 1.3. An object oscillating vertically on a spring has a pen attached to it. While the object is oscillating, a sheet of paper is moved perpendicular to the direction of motion of the spring, and the pen traces out the cosine curve in Equation 1.6.

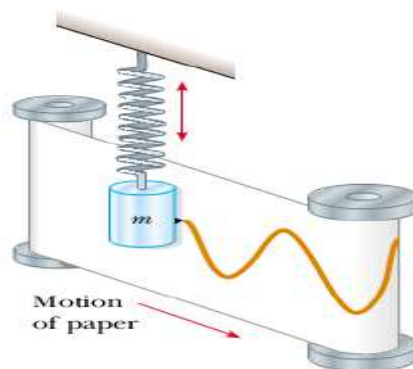


Figure 1.3 An experimental apparatus for demonstrating simple harmonic motion. A pen attached to the oscillating object traces out a sinusoidal pattern on the moving chart paper.

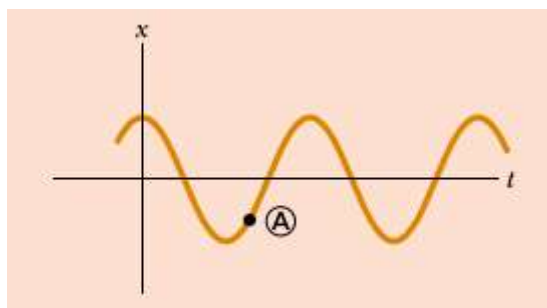


Figure 1.4 An x - t graph for an object undergoing simple harmonic motion. At a particular time, the object's position is indicated by (A) in the graph.

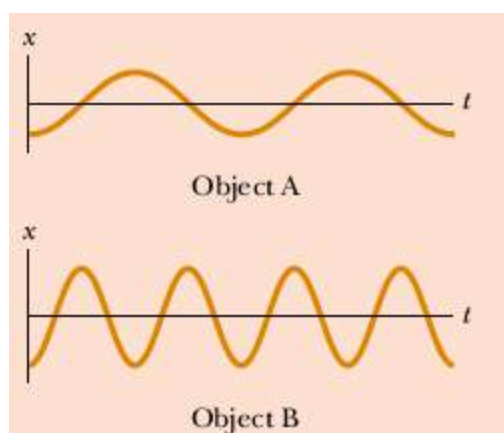


Figure 11.5 Two x - t graphs for objects undergoing simple harmonic motion. The amplitudes and frequencies are different for the two objects.

Let us investigate further the mathematical description of simple harmonic motion. The period T of the motion is the time interval required for the particle to go through one full cycle of its motion (Fig. 1.2a). That is, the values of x and v for the particle at time t equal the values of x and v at time $t + T$. We can relate the period to the angular frequency by using the fact that the phase increases by 2π radians in a time interval of T :

$$[\omega(t + T) + \phi] - (\omega t + \phi) = 2\pi$$

Simplifying this expression, we see that $\omega T = 2\pi$, or

$$T = \frac{2\pi}{\omega} \quad (1.10)$$

The inverse of the period is called the frequency f of the motion. Whereas the period is the time interval per oscillation, the frequency represents the number of oscillations that the particle undergoes per unit time interval:

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

The units of f are cycles per second, or hertz(Hz). Rearranging Equation 1.11 gives

$$\omega = 2\pi f = \frac{2\pi}{T} \quad (1.12)$$

We can use Equations 1.9, 1.10, and 1.11 to express the period and frequency of the motion for the particle–spring system in terms of the characteristics m and k of the system as

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}} \quad (1.13)$$

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (1.14)$$

That is, the period and frequency depend only on the mass of the particle and the force constant of the spring, and not on the parameters of the motion, such as A or ϕ . As we might expect, the frequency is larger for a stiffer spring (larger value of k) and decreases with increasing mass of the particle.

We can obtain the velocity and acceleration of a particle undergoing simple harmonic motion from Equations 1.7 and 1.8:

$$v \frac{dx}{dt} = -\omega A \sin(\omega t + \phi) \quad (1.15)$$

$$a \frac{d^2x}{dt^2} = -\omega^2 A \sin(\omega t + \phi) \quad (1.16)$$

From Equation 1.15 we see that, because the sine and cosine functions oscillate between ± 1 , the extreme values of the velocity v are $\pm \omega A$. Likewise, Equation 1.16 tells us that the extreme values of the acceleration a are $\pm \omega^2 A$. Therefore, the maximum values of the magnitudes of the velocity and acceleration are

$$v_{\max} = \omega A = \sqrt{\frac{k}{m}} A \quad (1.17)$$

$$a_{\max} = \omega^2 A = \frac{k}{m} A \quad (1.18)$$

Figure 1.6a plots position versus time for an arbitrary value of the phase constant. The associated velocity–time and acceleration–time curves are illustrated in Figures 1.6b and 1.6c. They show that the phase of the velocity differs from the phase of the position by $\pi/2$ rad, or 90° . That is, when x is a maximum or a minimum, the velocity is zero. Likewise, when x

is zero, the speed is a maximum. Furthermore, note that the phase of the acceleration differs from the phase of the position by π radians, or 180° . For example, when x is a maximum, a has a maximum magnitude in the opposite direction.

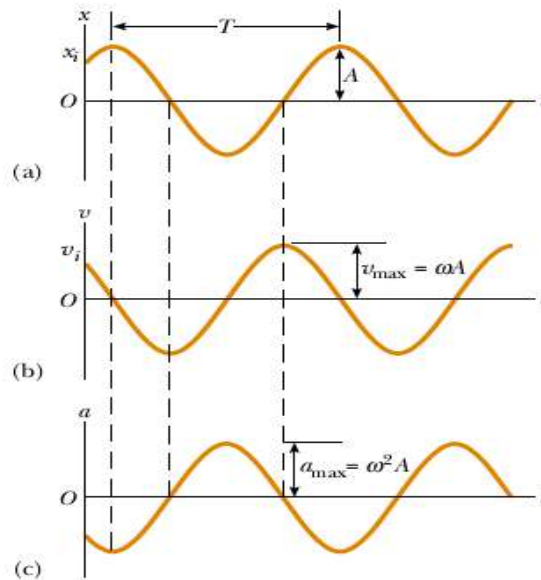


Figure 1.6 Graphical representation of simple harmonic motion. (a) Position versus time. (b) Velocity versus time. (c) Acceleration versus time. Note that at any specified time the velocity is 90° out of phase with the position and the acceleration is 180° out of phase with the position.

Equation 1.6 describes simple harmonic motion of a particle in general. Let us now see how to evaluate the constants of the motion. The angular frequency is evaluated using Equation 1.9. The constants A and ϕ (are evaluated from the initial conditions, that is, the state of the oscillator at $t = 0$).

Suppose we initiate the motion by pulling the particle from equilibrium by a distance A and releasing it from rest at $t = 0$, as in Figure 1.7. We must then require that

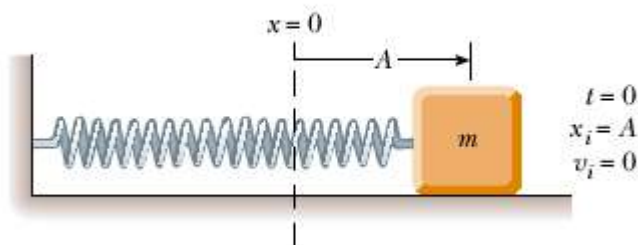


Figure 1.7 A block–spring system that begins its motion from rest with the block at $x=A$ at $t = 0$. In this case, $\phi = 0$ and thus $x= A \cos \omega t$.

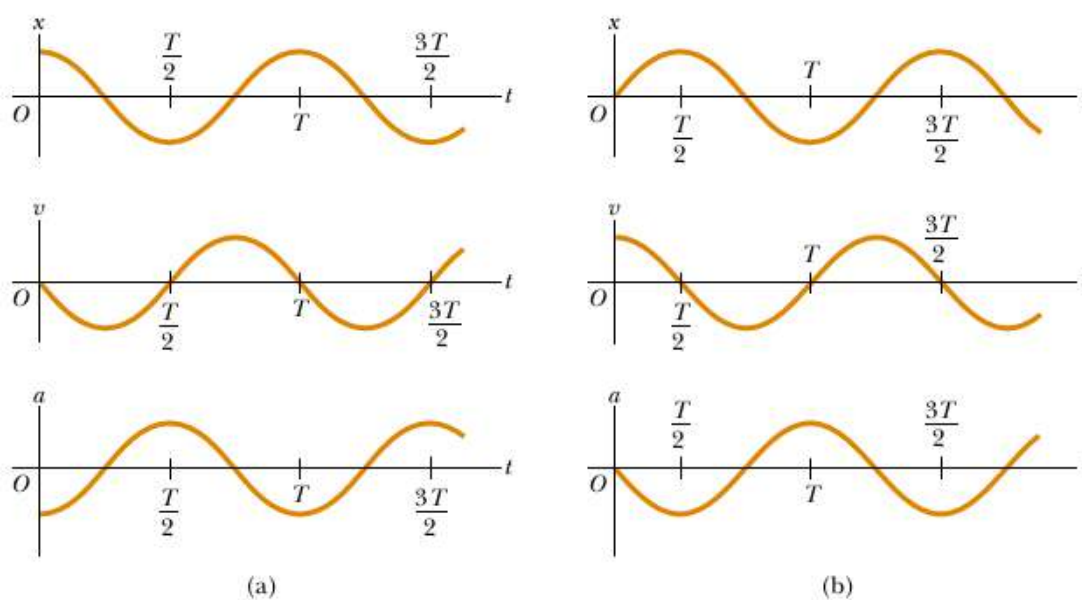


Figure 1.8 (a) Position, velocity, and acceleration versus time for a block undergoing simple harmonic motion under the initial conditions that at $t=0$, $x(0)=A$ and $v(0) = 0$. (b) Position, velocity, and acceleration versus time for a block undergoing simple harmonic motion under the initial conditions that at $t=0$, $x(0) = 0$ and $v(0)=v_i$.

our solutions for $x(t)$ and $v(t)$ (Eqs. 1.6 and 1.15) obey the initial conditions that $x(0) = A$ and $v(0) = 0$:

$$x(0) = A \cos \phi = A$$

$$v(0) = -\omega A \sin \phi = 0$$

These conditions are met if we choose $\phi = 0$, giving $x = A \cos \omega t$ as our solution. To check this solution, note that it satisfies the condition that $x(0) = A$, because $\cos 0 = 1$.

The position, velocity, and acceleration versus time are plotted in Figure 1.8a for this special case. The acceleration reaches extreme values of $\mp \omega^2 A$ when the position has extreme values of $\pm A$. Furthermore, the velocity has extreme values of $\pm \omega A$, which both occur at $x = 0$. Hence, the quantitative solution agrees with our qualitative description of this system.

Let us consider another possibility. Suppose that the system is oscillating and we define $t = 0$ as the instant that the particle passes through the unstretched position of the spring while moving to the right (Fig. 1.9). In this case we must require that our solutions for $x(t)$ and $v(t)$ obey the initial conditions that $x(0) = 0$ and $v(0) = v_i$:

$$x(0) = A \cos \phi = 0$$

$$v(0) = -\omega A \sin \phi = v_i$$

The first of these conditions tells us that $\phi = \pm \pi/2$. With these choices for ϕ , the second condition tells us that $A = \mp v_i/\omega$. Because the initial velocity is positive and the amplitude must be positive, we must have $\phi = -\pi/2$. Hence, the solution is given by

$$x = \frac{v_i}{\omega} \cos\left(\omega t - \frac{\pi}{2}\right)$$

The graphs of position, velocity, and acceleration versus time for this choice of $t = 0$ are shown in Figure 1.8b. Note that these curves are the same as those in Figure 1.8a, but shifted to the right by one fourth of a cycle. This is

described mathematically by the phase constant $\phi = -\pi/2$, which is one fourth of a full cycle of 2π .

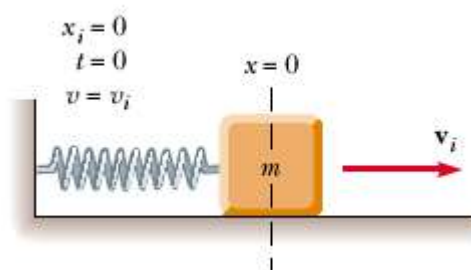


Figure 1.9 The block–spring system is undergoing oscillation, and $t = 0$ is defined at an instant when the block passes through the equilibrium position $x = 0$ and is moving to the right with speed v_i .

Example 1.1 An Oscillating Object

An object oscillates with simple harmonic motion along the x axis. Its position varies with time according to the equation where t is in seconds and the angles in the parentheses are in radians.

(A) Determine the amplitude, frequency, and period of the motion.

Solution: By comparing this equation with Equation 1.6, $x = A \cos(\omega t + \phi)$, we see that $A = 4.00$ m and ω rad/s. Therefore, $f = \omega/2\pi = 0.500$ Hz and $T = 1/f = 2.00$ s.

(B) Calculate the velocity and acceleration of the object at any time t .

Solution Differentiating x to find v , and v to find a , we obtain

$$\begin{aligned} v &= \frac{dx}{dt} = -(4.00 \text{ m/s}) \sin\left(\pi t + \frac{\pi}{4}\right) \frac{d}{dt}(\pi t) \\ &= -(4.00\pi \text{ m/s}) \sin\left(\pi t + \frac{\pi}{4}\right) \end{aligned}$$

$$\begin{aligned}
 v &= \frac{dv}{dt} = -(4.00\pi \text{ m/s}) \cos\left(\pi t + \frac{\pi}{4}\right) \frac{d}{dt}(\pi t) \\
 &= -(4.00\pi^2 \text{ m/s}^2) \cos\left(\pi t + \frac{\pi}{4}\right)
 \end{aligned}$$

(C) Using the results of part (B), determine the position, velocity, and acceleration of the object at $t = 1.00\text{s}$.

Solution Noting that the angles in the trigonometric functions are in radians, we obtain, at $t = 1.00 \text{ s}$.

$$\begin{aligned}
 x &= (4.00 \text{ m}) \cos\left(\pi + \frac{\pi}{4}\right) = (4.00\text{m}) \cos\left(\frac{5\pi}{4}\right) \\
 &= (4.00\text{m}) (-0.707) = -2.83\text{m}
 \end{aligned}$$

$$\begin{aligned}
 v &= -(4.00\pi \text{ m/s}) \sin\left(\frac{5\pi}{4}\right) \\
 &= -(4.00\pi \text{ m/s}) (-0.707) = 8.89\text{m/s}
 \end{aligned}$$

$$\begin{aligned}
 a &= -(4.00\pi^2 \text{ m/s}^2) \cos\left(\frac{5\pi}{4}\right) \\
 &= -(4.00\pi^2 \text{ m/s}^2) (-0.707) = 27.9\text{m/s}^2
 \end{aligned}$$

(D) Determine the maximum speed and maximum acceleration of the object.

Solution In the general expressions for v and a found in part (B), we use the fact that the maximum values of the sine and cosine functions are unity. Therefore, v varies between $\pm 4.00\pi \text{ m/s}$, and a varies between $\pm 4.00 \pi^2 \text{ m/s}^2$.

Thus,

$$v_{\max} = 4.00\pi \text{ m/s} = 12.6 \text{ m/s}$$

$$a_{\max} = 4.00\pi^2 \text{ m/s}^2 = 39.5 \text{ m/s}^2$$

We obtain the same results using the relations $v_{\max} = \omega A$ and $a_{\max} = \omega^2 A$, where $A = 4.00\text{m}$ and $\omega = \pi \text{ rad/s}$.

(E) Find the displacement of the object between $t = 0$ and $t = 1.00\text{s}$.

Solution The position at $t = 0$ is

In part (C), we found that the position at $t = 1.00\text{s}$ is -2.83m ; therefore, the displacement between $t = 0$ and $t = 1.00\text{s}$ is

$$\Delta x = x_f - x_i = -2.83 \text{ m} - 2.83 \text{ m} = -5.66 \text{ m}$$

Because the object's velocity changes sign during the first second, the magnitude of Δx is not the same as the distance traveled in the first second. (By the time the first second is over, the object has been through the point $x = -2.83\text{m}$ once, traveled to $x = -4.00\text{m}$, and come back to $x = -2.83\text{m}$.)

Example 1.2 Watch Out for Potholes!

A car with a mass of 1300kg is constructed so that its frame is supported by four springs. Each spring has a force constant of 20000N/m . If two people riding in the car have a combined mass of 160kg , find the frequency of vibration of the car after it is driven over a pothole in the road.

Solution To conceptualize this problem, think about your experiences with automobiles. When you sit in a car, it moves downward a small distance because your weight is compressing the springs further. If you push down on the front bumper and release, the front of the car oscillates a couple of times. We can model the car as being supported by a single spring and categorize this as an oscillation problem based on our simple spring model.

To analyze the problem, we first need to consider the effective spring constant of the four springs combined. For a given extension x of the springs, the combined force on the car is the sum of the forces from the individual springs:

$$F_{\text{total}} = \Sigma(-kx) = -(\Sigma k)x$$

where x has been factored from the sum because it is the same for all four springs. We see that the effective spring constant for the combined springs is the sum of the individual spring constants:

$$k_{\text{eff}} = \Sigma k = 4 \times 20\,000 \text{ N/m} = 80\,000 \text{ N/m}$$

Hence, the frequency of vibration is, from Equation 1.14,

$$f = \frac{1}{2\pi} \sqrt{\frac{k_{\text{eff}}}{m}} = \frac{1}{2\pi} \sqrt{\frac{80\,000 \text{ N/m}}{1\,460 \text{ kg}}} = 1.18 \text{ Hz}$$

To finalize the problem, note that the mass we used here is that of the car plus the people, because this is the total mass that is oscillating. Also note that we have explored only up-and-down motion of the car. If an oscillation is established in which the car rocks back and forth such that the front end goes up when the back end goes down, the frequency will be different.

What If? Suppose the two people exit the car on the side of the road. One of them pushes downward on the car and releases it so that it oscillates vertically. Is the frequency of the oscillation the same as the value we just calculated?

Answer The suspension system of the car is the same, but the mass that is oscillating is smaller—it no longer includes the mass of the two people. Thus, the frequency should be higher. Let us calculate the new frequency:

$$f = \frac{1}{2\pi} \sqrt{\frac{k_{\text{eff}}}{m}} = \frac{1}{2\pi} \sqrt{\frac{80\,000 \text{ N/m}}{1\,300 \text{ kg}}} = 1.25 \text{ Hz}$$

As we predicted conceptually, the frequency is a bit higher.

Example 1.3 A Block–Spring System

A 200-g block connected to a light spring for which the force constant is 5.00 N/m is free to oscillate on a horizontal, frictionless surface. The block is displaced 5.00 cm from equilibrium and released from rest, as in Figure 1.7.

(A) Find the period of its motion.

Solution: From Equations 1.9 and 1.10, we know that the angular frequency of a block–spring system is and the period is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.00 \text{ N/m}}{200 \times 10^{-3} \text{ kg}}} = 5.00 \text{ rad/s}$$

and the period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00 \text{ rad/s}} = 1.26 \text{ s}$$

(B) Determine the maximum speed of the block.

Solution: We use Equation 1.17:

$$v_{\text{max}} = \omega A = (5.00 \text{ rad/s}) (5.00 \times 10^{-2} \text{ m}) = 0.250 \text{ m/s}$$

(C) What is the maximum acceleration of the block?

Solution: We use Equation 1.18:

$$a_{\text{max}} = \omega^2 A = (5.00 \text{ rad/s})^2 (5.00 \times 10^{-2} \text{ m}) = 1.25 \text{ m/s}^2$$

(D) Express the position, speed, and acceleration as functions of time.

Solution: We find the phase constant from the initial condition that $x = A$ at $t = 0$:

$$x(0) = A \cos \phi = A$$

which tells us that $\phi = 0$. Thus, our solution is $x = A \cos \omega t$. Using this expression and the results from (A), (B), and (C), we find that

$$x = A \cos \omega t = (0.050 \text{ m/s}) \cos 5.00t$$

$$v = \omega A \cos \omega t = - (0.250 \text{ m/s}) \sin 5.00t$$

$$a = -\omega^2 A \cos \omega t = - (1.25 \text{ m/s}^2) \cos 5.00t$$

What If? What if the block is released from the same initial position, $x_i = 5.00 \text{ cm}$, but with an initial velocity of $v_i = 0.100 \text{ m/s}$? Which parts of the solution change and

what are the new answers for those that do change?

Answers: Part (A) does not change—the period is independent of how the oscillator is set into motion. Parts (B), (C), and (D) will change. We begin by considering position and velocity expressions for the initial conditions:

$$(1) \quad x(0) = A \cos \phi = x_i$$

$$(2) \quad v(0) = -\omega A \sin \phi = v_i$$

Dividing Equation (2) by Equation (1) gives us the phase constant:

$$\frac{-\omega A \sin \phi}{A \cos \phi} = \frac{v_i}{x_i}$$

$$\tan \phi = \frac{v_i}{\omega x_i} = -\frac{0.100 \text{ m}}{(5.00 \text{ rad/s})(0.0500 \text{ m})} = 0.400$$

$$\phi = 0.12 \pi$$

Now, Equation (1) allows us to find A:

$$A = \frac{x_i}{\cos \phi} = \frac{0.0500 \text{ m}}{\cos(0.12\pi)} = 0.0539 \text{ m}$$

The new maximum speed is

$$v_{\max} = \omega A = (5.00 \text{ rad/s}) (5.39 \times 10^{-2} \text{ m}) = 0.269 \text{ m/s}$$

The new magnitude of the maximum acceleration is

$$a_{\max} = \omega^2 A = (5.00 \text{ rad/s})^2 (5.39 \times 10^{-2} \text{ m}) = 1.35 \text{ m/s}^2$$

The new expressions for position, velocity, and acceleration are

$$x = (0.0539 \text{ m}) \cos (5.00t + 0.12\pi)$$

$$v = -(0.269 \text{ m/s}) \sin (5.00t + 0.12\pi)$$

$$a = -(1.35 \text{ m/s}^2) \cos (5.00t + 0.12\pi)$$

As we saw in Chapters 7 and 8, many problems are easier to solve with an energy approach rather than one based on variables of motion. This particular What If? is easier to solve from an energy approach. Therefore, in the next section we shall investigate the energy of the simple harmonic oscillator.

1.3 Energy of the Simple Harmonic Oscillator

Let us examine the mechanical energy of the block–spring system illustrated in Figure 1.1. Because the surface is frictionless, we expect the total mechanical energy of the system to be constant. We assume a massless spring, so the kinetic energy of the system corresponds only to that of the

block. We can use Equation 1.15 to express the kinetic energy of the block as

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi) \quad (1.19)$$

The elastic potential energy stored in the spring for any elongation x is given by $\frac{1}{2}kx^2$. Using Equation 1.6, we obtain

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi) \quad (1.20)$$

We see that K and U are always positive quantities. Because $\omega^2 = k/m$, we can express the total mechanical energy of the simple harmonic oscillator as

$$E = K + U = \frac{1}{2}kA^2[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

From the identity $\sin^2 \theta + \cos^2 \theta = 1$, we see that the quantity in square brackets is unity. Therefore, this equation reduces to

$$E = \frac{1}{2}kA^2 \quad (1.21)$$

That is, the total mechanical energy of a simple harmonic oscillator is a constant of the motion and is proportional to the square of the amplitude. Note that U is small when K is large, and vice versa, because the sum must be constant. In fact, the total mechanical energy is equal to the maximum potential energy stored in the spring when $x = \pm A$ because $v = 0$ at these points and thus there is no kinetic energy. At the equilibrium position, where $U = 0$ because $x = 0$, the total energy, all in the form of kinetic energy, is again $\frac{1}{2}kA^2$. That is,

$$E = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}m \frac{k}{m} A^2 = \frac{1}{2}kA^2 \quad (\text{at } x = 0)$$

Plots of the kinetic and potential energies versus time appear in Figure 1.10a, where we have taken $\phi = 0$. As already mentioned, both K and U are always positive, and at all times their sum is a constant equal to $\frac{1}{2}kA^2$, the total energy of the system. The variations of K and U with the position x of the block are plotted in Figure 1.10b.

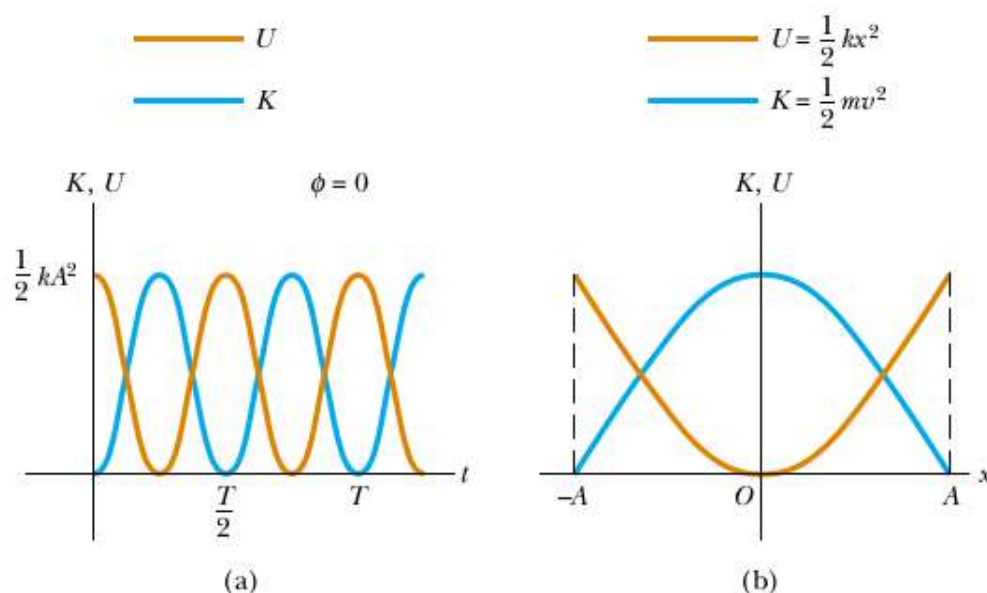


Figure 1.10(a) Kinetic energy and potential energy versus time for a simple harmonic oscillator with $\phi = 0$. (b) Kinetic energy and potential energy versus position for a simple harmonic oscillator. In either plot, note that $K + U = \text{constant}$.

Energy is continuously being transformed between potential energy stored in the spring and kinetic energy of the block.

Figure 1.11 illustrates the position, velocity, acceleration, kinetic energy, and potential energy of the block–spring system for one full period of the motion. Most of the ideas discussed so far are incorporated in this important figure. Study it carefully.

Finally, we can use the principle of conservation of energy to obtain the velocity for an arbitrary position by expressing the total energy at some arbitrary position x as

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$v = \pm \sqrt{\frac{k}{m}(A^2 - x^2)} = \pm \omega \sqrt{A^2 - x^2} \quad (1.22)$$

When we check Equation 1.22 to see whether it agrees with known cases, we find that it verifies the fact that the speed is a maximum at $x = 0$ and is zero at the turning points $x = \pm A$.

You may wonder why we are spending so much time studying simple harmonic oscillators. We do so because they are good models of a wide variety of physical phenomena. For example, recall the Lennard–Jones potential. This complicated function describes the forces holding atoms together. Figure 1.12a shows that, for small displacements from the equilibrium position, the potential energy curve for this function approximates a parabola, which represents the potential energy function for a simple harmonic oscillator. Thus, we can model the complex atomic binding forces as being due to tiny springs, as depicted in Figure 1.12b.

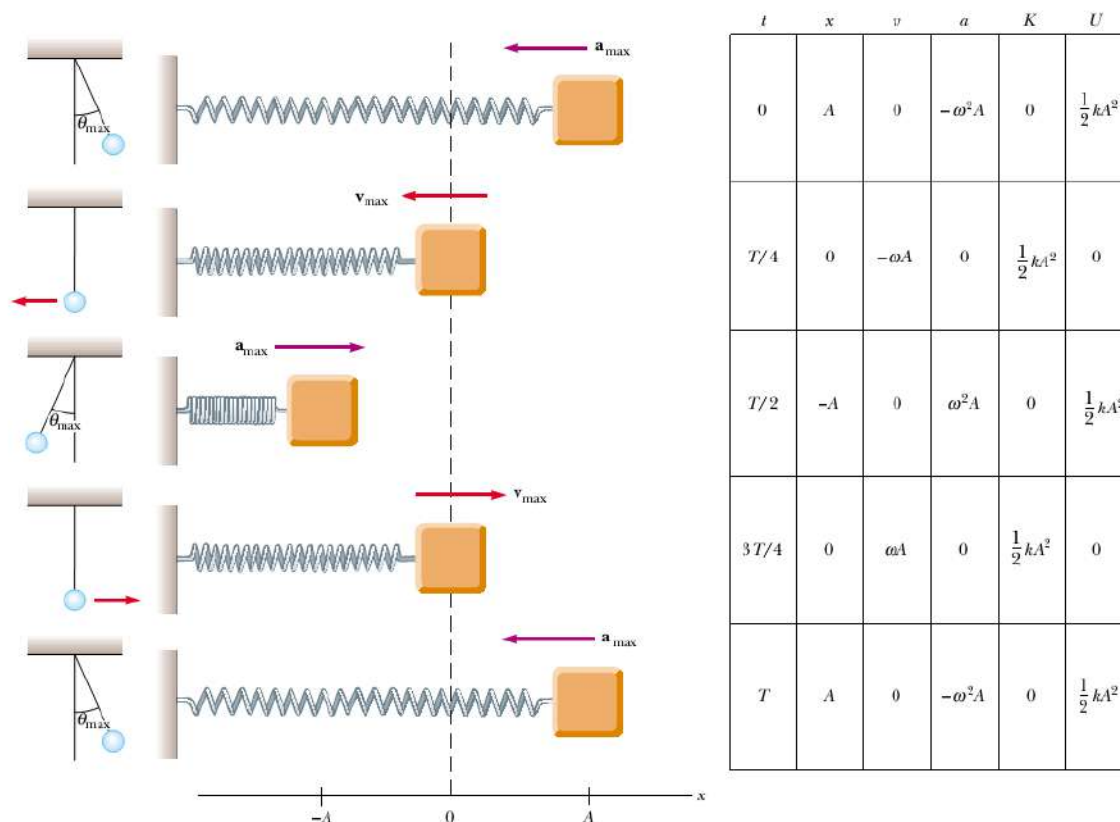


Figure 1.11 Simple harmonic motion for a block–spring system and its analogy to the motion of a simple pendulum (Section 1.5). The parameters in the table at the right refer to the block–spring system, assuming that at $t = 0$, $x = A$ so that $x = A \cos \omega t$.

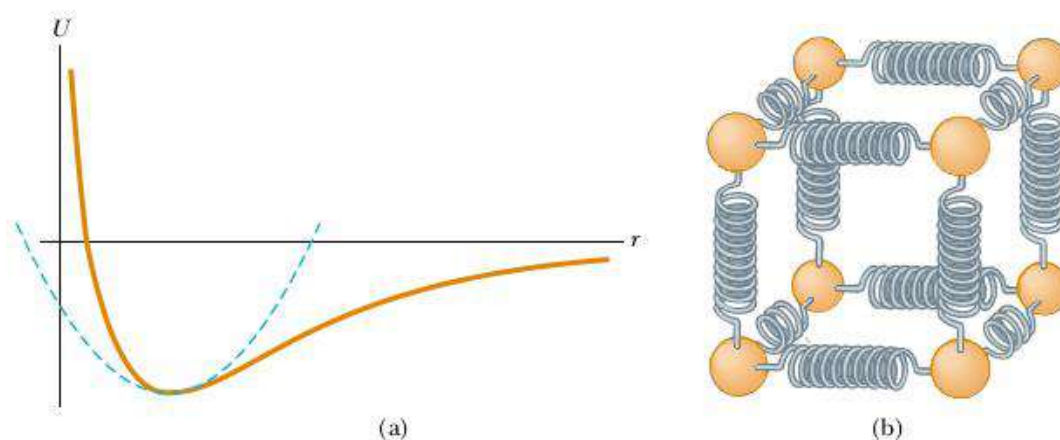


Figure 1.12(a) If the atoms in a molecule do not move too far from their equilibrium positions, a graph of potential energy versus separation distance between atoms is similar to the graph of potential energy versus position for a simple harmonic oscillator (blue curve). (b) The forces between atoms in a solid can be modeled by imagining springs between neighboring atoms.

The ideas presented in this chapter apply not only to block–spring systems and atoms, but also to a wide range of situations that include bungee jumping, tuning in a television station, and viewing the light emitted by a laser. You will see more examples of simple harmonic oscillators as you work through this book.

Example 1.4 Oscillations on a Horizontal Surface

A 0.500-kg cart connected to a light spring for which the force constant is 20.0 N/m oscillates on a horizontal, frictionless air track.

(A) Calculate the total energy of the system and the maximum speed of the cart if the amplitude of the motion is 3.00 cm.

Solution: Using Equation 1.21, we obtain

$$\begin{aligned} E &= \frac{1}{2} kA^2 = \frac{1}{2} (20.0 \text{ N/m}) (3.00 \times 10^{-2} \text{ m})^2 \\ &= 9.00 \times 10^{-3} \text{ J} \end{aligned}$$

When the cart is located at $x = 0$, we know that $U = 0$ and $E = \frac{1}{2} mv_{\text{max}}^2$; therefore

$$\begin{aligned} \frac{1}{2} mv_{\text{max}}^2 &= 9.00 \times 10^{-3} \text{ J} \\ v_{\text{max}} &= \sqrt{\frac{2(9.00 \times 10^{-3} \text{ J})}{0.500 \text{ kg}}} = 0.190 \text{ m/s} \end{aligned}$$

(B) What is the velocity of the cart when the position is 2.00 cm?

Solution: We can apply Equation 1.22 directly

$$v = \pm \sqrt{\frac{k}{m}(A^2 - x^2)}$$

$$= \pm \sqrt{\frac{20.0 \text{ N/m}}{0.500 \text{ kg}} [(0.0300 \text{ m})^2 - (0.0200 \text{ m})^2]}$$

$$= \pm 0.141 \text{ m/s}$$

The positive and negative signs indicate that the cart could be moving to either the right or the left at this instant.

(C) Compute the kinetic and potential energies of the system when the position is 2.00cm.

Solution: Using the result of (B), we find that

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.500 \text{ kg})(0.141 \text{ m/s})^2 = 5.00 \times 10^{-3} \text{ J}$$

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(20.0 \text{ N/m})(0.0200 \text{ m})^2 = 4.00 \times 10^{-3} \text{ J}$$

Note that $K + U = E$.

What If? The motion of the cart in this example could have been initiated by releasing the cart from rest at $x = 3.00 \text{ cm}$. What if the cart were released from the same position, but with an initial velocity of $v = -0.100 \text{ m/s}$? What are the new amplitude and maximum speed of the cart?

Answer: This is the same type of question as we asked at the end of Example 1.3, but here we apply an energy approach. First let us calculate the total energy of the system at $t = 0$, which consists of both kinetic energy and potential energy:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}(0.500 \text{ kg})(-0.100 \text{ m/s})^2 + \frac{1}{2}(20.0 \text{ N/m})(0.0300 \text{ m})^2$$

$$= 1.5 \times 10^{-2} \text{ J}$$

To find the new amplitude, we equate this total energy to the potential energy when the cart is at the end point of the motion:

$$E = \frac{1}{2} kA^2$$

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(1.15 \times 10^{-2} \text{ J})}{20.0 \text{ N/m}}} = 0.0339 \text{ m}$$

Note that this is larger than the previous amplitude of 0.0300 m. To find the new maximum speed, we equate this total energy to the kinetic energy when the cart is at the equilibrium position:

$$E = \frac{1}{2} mv_{\text{max}}^2$$

$$v_{\text{max}} = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(1.15 \times 10^{-2} \text{ J})}{0.500 \text{ N/m}}} = 0.214 \text{ m/s}$$

This is larger than the value found in part (a) as expected because the cart has an initial velocity at $t = 0$.

1.4 Wave motion and propagation of a disturbance

Most of us experienced waves as children when we dropped a pebble into a pond. At the point where the pebble hits the water's surface, waves are created. These waves move outward from the creation point in expanding circles until they reach the shore. If you were to examine carefully the motion of a beach ball floating on the disturbed water, you would see that the ball moves vertically and horizontally about its original position but does not undergo any net displacement away from or toward the point where the pebble hit the water. The small elements of water in contact with the beach ball, as well as all the other water elements on the pond's surface, behave in

the same way. That is, the water wave moves from the point of origin to the shore, but the water is not carried with it.

The world is full of waves, the two main types being mechanical waves and electromagnetic waves. In the case of mechanical waves, some physical medium is being disturbed—in our pebble and beach ball example, elements of water are disturbed. Electromagnetic waves do not require a medium to propagate; some examples of electromagnetic waves are visible light, radio waves, television signals, and x-rays. Here, in this part of the book, we study only mechanical waves.

The wave concept is abstract. When we observe what we call a water wave, what we see is a rearrangement of the water's surface. Without the water, there would be no wave. A wave traveling on a string would not exist without the string. Sound waves could not travel from one point to another if there were no air molecules between the two points. With mechanical waves, what we interpret as a wave corresponds to the propagation of a disturbance through a medium.

Considering further the beach ball floating on the water, note that we have caused the ball to move at one point in the water by dropping a pebble at another location. The ball has gained kinetic energy from our action, so energy must have transferred from the point at which we drop the pebble to the position of the ball. This is a central feature of wave motion—energy is transferred over a distance, but matter is not.

All waves carry energy, but the amount of energy transmitted through a medium and the mechanism responsible for that transport of energy differ from case to case. For instance, the power of ocean waves during a storm is

much greater than the power of sound waves generated by a single human voice.

In the introduction, we alluded to the essence of wave motion—the transfer of energy through space without the accompanying transfer of matter. In the list of energy transfer mechanisms, two mechanisms depend on waves—mechanical waves and electromagnetic radiation. By contrast, in another mechanism—matter transfer—the energy transfer is accompanied by a movement of matter through space.

All mechanical waves require (1) some source of disturbance, (2) a medium that can be disturbed, and (3) some physical mechanism through which elements of the medium can influence each other. One way to demonstrate wave motion is to flick one end of a long rope that is under tension and has its opposite end fixed, as shown in Figure 2.1. In this manner, a single bump (called a pulse) is formed and travels along the rope with a definite speed. Figure 2.1 represents four consecutive “snapshots” of the creation and propagation of the traveling pulse. The rope is the medium through which the pulse travels. The pulse has a definite height and a definite speed of propagation along the medium (the rope). As we shall see later, the properties of this particular medium that determine the speed of the disturbance are the tension in the rope and its mass per unit length. The shape of the pulse changes very little as it travels along the rope.

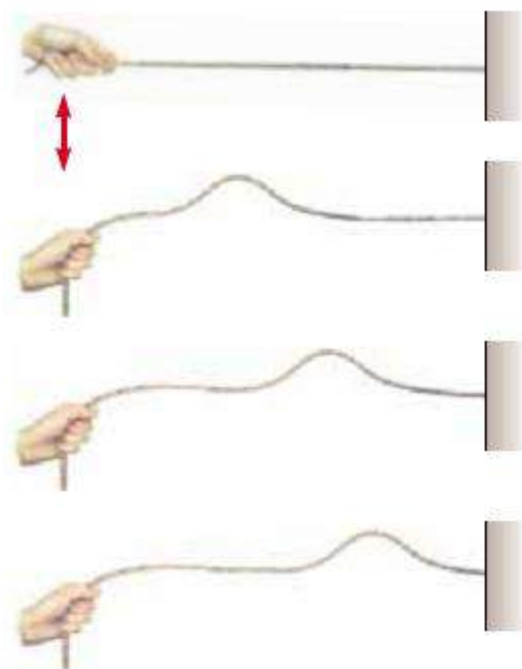


Figure 1.13 A pulse traveling down a stretched rope. The shape of the pulse is approximately un-changed as it travels along the rope.

We shall first focus our attention on a pulse traveling through a medium. Once we have explored the behavior of a pulse, we will then turn our attention to a wave, which is a periodic disturbance traveling through a medium. We created a pulse on our rope by flicking the end of the rope once, as in Figure 1.13. If we were to move the end of the rope up and down repeatedly, we would create a traveling wave, which has characteristics that a pulse does not have.

As the pulse in Figure 1.13 travels, each disturbed element of the rope moves in a direction perpendicular to the direction of propagation. Figure 1.14 illustrates this point for one particular element, labeled P. Note that no part of the rope ever moves in the direction of the propagation.

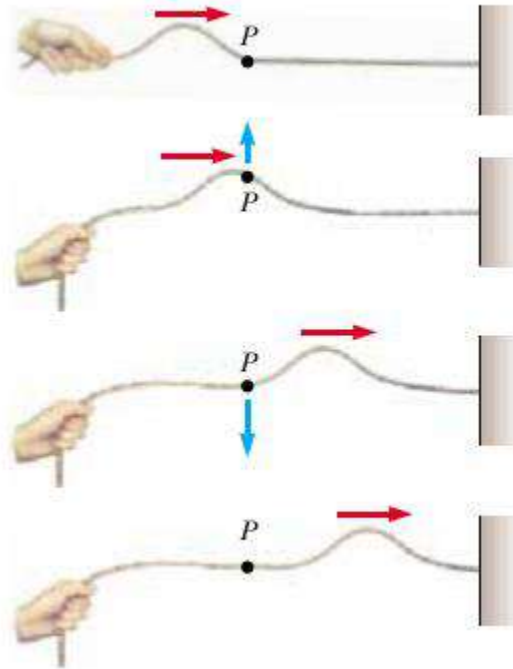


Figure 1.14 A transverse pulse traveling on a stretched rope. The direction of motion of any element P of the rope (blue arrows) is perpendicular to the direction of propagation (red arrows).

Compare this with another type of pulse—one moving down a long, stretched spring, as shown in Figure 1.15. The left end of the spring is pushed briefly to the right and then pulled briefly to the left. This movement creates a sudden compression of a region of the coils. The compressed region travels along the spring (to the right in Figure 1.15). The compressed region is followed by a region where the coils are extended. Notice that the direction of the displacement of the coils is parallel to the direction of propagation of the compressed region.

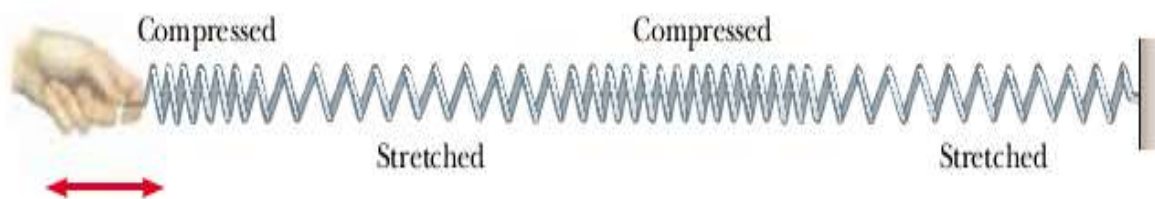


Figure 1.15 A longitudinal pulse along a stretched spring. The displacement of the coils is parallel to the direction of the propagation.

Sound waves, which we shall discuss in Chapter 2, are another example of longitudinal waves. The disturbance in a sound wave is a series of high-pressure and low-pressure regions that travel through air.

Some waves in nature exhibit a combination of transverse and longitudinal displacements. Surface water waves are a good example. When a water wave travels on the surface of deep water, elements of water at the surface move in nearly circular paths, as shown in Figure 1.16. Note that the disturbance has both transverse and longitudinal components. The transverse displacements seen in Figure 1.16 represent the variations in vertical position of the water elements. The longitudinal displacement can be explained as follows: as the wave passes over the water's surface, water elements at the highest points move in the direction of propagation of the wave, whereas elements at the lowest points move in the direction opposite the propagation.

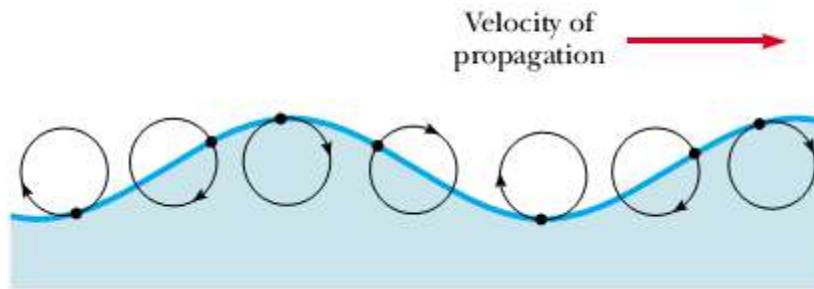


Figure 1.16 The motion of water elements on the surface of deep water in which a wave is propagating is a combination of transverse and longitudinal displacements, with the result that elements at the surface move in nearly circular paths. Each element is displaced both horizontally and vertically from its equilibrium position.

Sinusoidal Waves

In this section, we introduce an important wave function whose shape is shown in Figure 1.17. The wave represented by this curve is called a sinusoidal wave because the curve is the same as that of the function $\sin \theta$ plotted against θ . On a rope, a sinusoidal wave could be established by shaking the end of the rope up and down in simple harmonic motion.

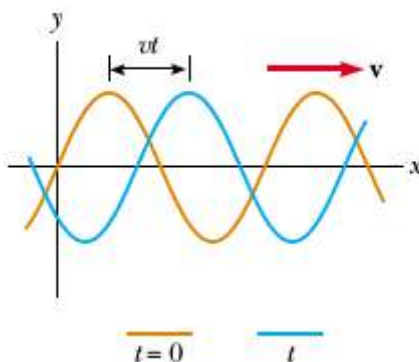


Figure 1.17 A one-dimensional sinusoidal wave traveling to the right with a speed v . The brown curve represents a snapshot of the wave at $t = 0$, and the blue curve represents a snapshot at some later time t .

Figure 1.18a shows a snapshot of a wave moving through a medium. Figure 1.18b shows a graph of the position of one element of the medium as a function of time. The point at which the displacement of the element from its normal position is highest is called the crest of the wave. The distance from one crest to the next is called the wavelength λ (Greek lambda). More generally, the wavelength is the minimum distance between any two identical points (such as the crests) on adjacent waves, as shown in Figure 1.18a.

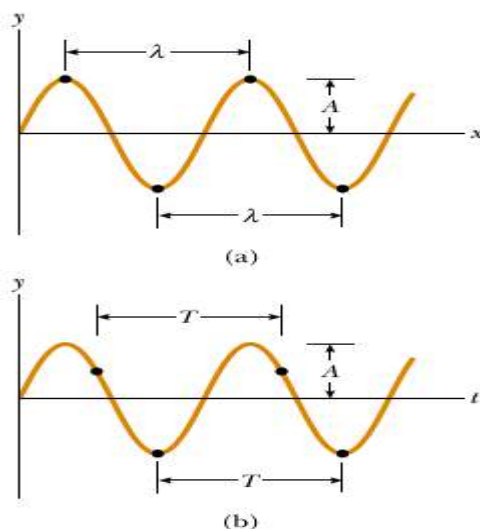


Figure 1.18 (a) The wave-length λ of a wave is the distance between adjacent crests or adjacent troughs. (b) The period T of a wave is the time interval required for the wave to travel one wavelength.

Consider the sinusoidal wave in Figure 1.18a, which shows the position of the wave at $t = 0$. Because the wave is sinusoidal, we expect the wave function at this instant to be expressed as $y(x, 0) = A \sin ax$, where A is the amplitude and a is a constant to be determined. At $x = 0$, we see that $y(0, 0) = A \sin a(0) = 0$, consistent with Figure 1.18a. The next value of x for which y is zero is $x = \lambda/2$. Thus

$$y\left(\frac{\lambda}{2}, 0\right) = A \sin a\left(\frac{\lambda}{2}\right) = 0$$

For this to be true, we must have $a(\lambda/2) = \pi$, or $a = 2\pi/\lambda$. Thus, the function describing the positions of the elements of the medium through which the sinusoidal wave is traveling can be written

$$y(x, 0) = A \sin\left(\frac{2\pi}{\lambda} x\right) \quad (1.23)$$

where the constant A represents the wave amplitude and the constant λ is the wavelength. We see that the vertical position of an element of the medium is the same whenever x is increased by an integral multiple of λ . If the wave moves to the right with a speed v , then the wave function at some later time t is

$$y(x, t) = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right] \quad (1.24)$$

That is, the traveling sinusoidal wave moves to the right a distance vt in the time t , as shown in Figure 1.17. Note that the wave function has the form $f(x - vt)$ (Eq. 2.1). If the wave were traveling to the left, the quantity $x - vt$ would be replaced by $x + vt$.

By definition, the wave travels a distance of one wavelength in one period T . Therefore, the wave speed, wavelength, and period are related by the expression.

$$v = \frac{\lambda}{T} \quad (1.25)$$

Substituting this expression for v into Equation 1.24, we find that

$$y = A \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] \quad (1.26)$$

This form of the wave function shows the periodic nature of y . (We will often use y rather than $y(x, t)$ as a shorthand notation.) At any given time t , y has the same value at the positions x , $x + \lambda$, $x + 2\lambda$, and so on. Furthermore, at any given position x , the value of y is the same at times t , $t + T$, $t + 2T$, and so on.

We can express the wave function in a convenient form by defining two other quantities, the angular wave number k (usually called simply the wave number) and the angular frequency ω :

$$k \equiv \frac{2\pi}{\lambda} \quad (1.27)$$

$$\omega \equiv \frac{2\pi}{T} \quad (1.28)$$

Using these definitions, we see that Equation 1.27 can be written in the more compact form

$$y = A \sin (kx - \omega t) \quad (1.29)$$

Using Equations 1.27, and 1.28, we can express the wave speed v originally given in Equation 1.25 in the alternative forms

$$v = \frac{\omega}{k} \quad (1.30)$$

$$v = \lambda f \quad (1.31)$$

The wave function given by Equation 1.29 assumes that the vertical position y of an element of the medium is zero at $x = 0$ and $t = 0$. This need not be the case. If it is not, we generally express the wave function in the form

$$y = A \sin (kx - \omega t + \phi) \quad (1.32)$$

where ϕ is the phase constant. This constant can be determined from the initial conditions.

Chapter 2

Sound Waves

Sound waves are the most common example of longitudinal waves. They travel through any material medium with a speed that depends on the properties of the medium. As the waves travel through air, the elements of air vibrate to produce changes in density and pressure along the direction of motion of the wave. If the source of the sound waves vibrates sinusoidally, the pressure variations are also sinusoidal. The mathematical description of sinusoidal sound waves is very similar to that of sinusoidal string waves, which were discussed in the previous chapter.

Sound waves are divided into three categories that cover different frequency ranges. (1) Audible waves lie within the range of sensitivity of the human ear. They can be generated in a variety of ways, such as by musical instruments, human voices, or loudspeakers. (2) Infrasonic waves have frequencies below the audible range. Elephants can use infrasonic waves to communicate with each other, even when separated by many kilometers. (3) Ultrasonic waves have frequencies above the audible range. You may have used a “silent” whistle to retrieve your dog. The ultrasonic sound it emits is easily heard by dogs, although humans cannot detect it at all. Ultrasonic waves are also used in medical imaging.

We begin this chapter by discussing the speed of sound waves and then wave intensity, which is a function of wave amplitude. We then provide an alternative description of the intensity of sound waves that compresses the wide range of intensities to which the ear is sensitive into a smaller range

for convenience. We investigate the effects of the motion of sources and/or listeners on the frequency of a sound. Finally, we explore digital reproduction of sound, focusing in particular on sound systems used in modern motion pictures.

3.1 Speed of Sound Waves

Let us describe pictorially the motion of a one-dimensional longitudinal pulse moving through a long tube containing a compressible gas. A piston at the left end can be moved to the right to compress the gas and create the pulse. Before the piston is moved, the gas is undisturbed and of uniform density, as represented by the uniformly shaded region in Figure 2.1a. When the piston is suddenly pushed to the right (Fig. 2.1b), the gas just in front of it is compressed (as represented by the more heavily shaded region); the pressure and density in this region are now higher than they were before the piston moved. When the piston comes to rest (Fig. 2.1c), the compressed region of the gas continues to move to the right, corresponding to a longitudinal pulse traveling through the tube with speed v . Note that the piston speed does not equal v . Furthermore, the compressed region does not “stay with” the piston as the piston moves, because the speed of the wave is usually greater than the speed of the piston.

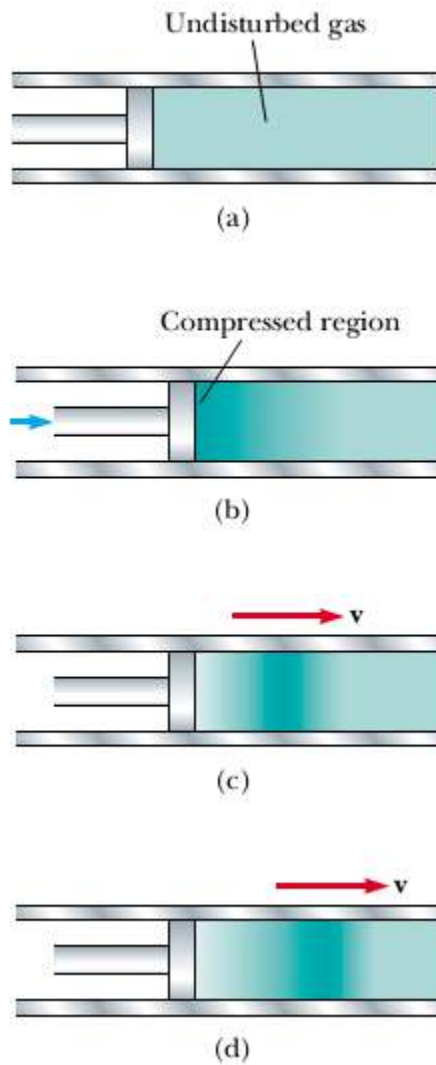


Figure 2.1 Motion of a longitudinal pulse through a compressible gas. The compression (darker region) is produced by the moving piston.

The speed of sound waves in a medium depends on the compressibility and density of the medium. If the medium is a liquid or a gas and has a bulk modulus B and density ρ , the speed of sound waves in that medium is

$$v = \sqrt{\frac{B}{\rho}} \quad (2.1)$$

The wave speed depends on an elastic property of the medium—bulk modulus B or string tension T —and on an inertial property of the medium—

ρ or μ . In fact, the speed of all mechanical waves follows an expression of the general form

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

For longitudinal sound waves in a solid rod of material, for example, the speed of sound depends on Young's modulus Y and the density ρ . Table 2.1 provides the speed of sound in several different materials.

The speed of sound also depends on the temperature of the medium. For sound traveling through air, the relationship between wave speed and medium temperature is

$$v = (331 \text{ m/s}) \sqrt{\frac{1 + T_C}{273^\circ \text{C}}}$$

where 331 m/s is the speed of sound in air at 0°C , and T_C is the air temperature in degrees Celsius. Using this equation, one finds that at 20°C the speed of sound in air is approximately 343m/s.

This information provides a convenient way to estimate the distance to a thunder-storm. You count the number of seconds between seeing the flash of lightning and hearing the thunder. Dividing this time by 3 gives the approximate distance to the lightning in kilometers, because 343 m/s is approximately 1/3 km/s. Dividing the time in seconds by 5 gives the approximate distance to the lightning in miles, because the speed of sound in ft/s (1125ft/s) is approximately $\frac{1}{5}$ mi/s.

Table 2.1 Speed of sound various media.

Medium	v(m/s)	Medium	v(m/s)	Medium	v(m/s)
Gases		Liquids at 25°C		Solids ^a	
Hydrogen (0°C)	1286	Glycerol	1904	Pyrex glass	5640
Helium (0°C)	972	Seawater	1533	Iron	5950
Air (20°C)	343	Water	1493	Aluminum	6420
Air (0°C)	331	Mercury	1450	Brass	4700
Oxygen (0°C)	317	Kerosene	1324	Copper	5010
		Methyl alcohol	1143	Gold	3240
		Carbon tetrachloride	926	Lucite	2680
				Lead	1960
				Rubber	1600

^aValues given are for propagation of longitudinal waves in bulk media. Speeds for longitudinal waves in thin rods are smaller, and speeds of trans-verse waves in bulk are smaller yet.

Example 2.1 Speed of Sound in a Liquid

(A) Find the speed of sound in water, which has a bulk modulus of 2.1×10^9 N/m² at a temperature of 0°C and a density of 1.00×10^3 kg/m³.

Solution: Using Equation 2.1, we find that

$$v_{\text{water}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.1 \times 10^9 \text{ N/m}^2}{1.00 \times 10^3 \text{ kg/m}^3}} = 1.4 \text{ km/s}$$

In general, sound waves travel more slowly in liquids than in solids because liquids are more compressible than solids. Note that the speed of sound in water is lower at 0°C than at 25°C (Table 2.1).

(B) Dolphins use sound waves to locate food. Experiments have shown that a dolphin can detect a 7.5-cm target 110m away, even in murky water. For a bit of “dinner” at that distance, how much time passes between the moment the dolphin emits a sound pulse and the moment the dolphin hears its reflection and thereby detects the distant target?

Solution: The total distance covered by the sound wave as it travels from dolphin to target and back is $2 \times 110 \text{ m} = 220\text{m}$. we have, for 25°C water

$$\Delta t = \frac{\Delta x}{v_x} = \frac{220\text{m}}{1.533\text{m/s}} = 0.14\text{s}$$

2.2 Periodic Sound Waves

This section will help you better comprehend the nature of sound waves. An important fact for understanding how our ears work is that pressure variations control what we hear.

One can produce a one-dimensional periodic sound wave in a long, narrow tube containing a gas by means of an oscillating piston at one end, as shown in Figure 2.2. The darker parts of the colored areas in this figure represent regions where the gas is com-pressed and thus the density and pressure are above their equilibrium values. A com-pressed region is formed whenever the piston is pushed into the tube. This compressed region, called a compression, moves through the tube as a pulse, continuously com-pressing the region just in front of itself. When the piston is pulled back, the gas in front of it expands, and the pressure and density in this region fall below their equilibrium values (represented by the lighter parts of the colored areas in Fig. 2.2). These low-pressure regions, called rarefactions, also propagate along the tube, following the compressions. Both regions move with a speed equal to the speed of sound in the medium.

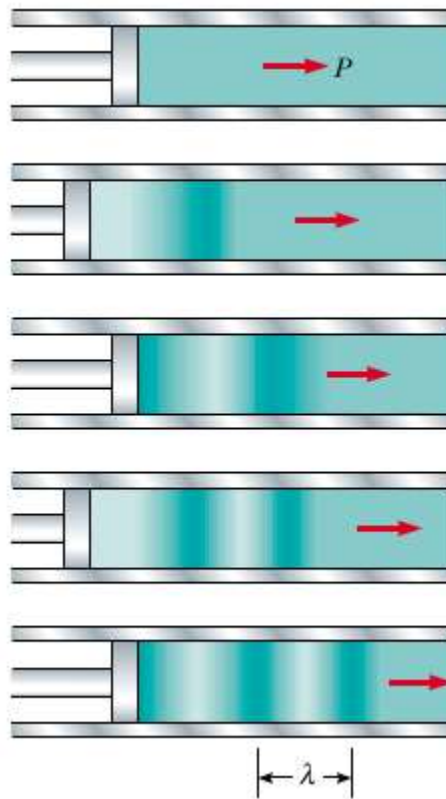


Figure 2.2 A longitudinal wave propagating through a gas-filled tube. The source of the wave is an oscillating piston at the left.

As the piston oscillates sinusoidally, regions of compression and rarefaction are continuously set up. The distance between two successive compressions (or two successive rarefactions) equals the wavelength λ . As these regions travel through the tube, any small element of the medium moves with simple harmonic motion parallel to the direction of the wave. If $s(x, t)$ is the position of a small element relative to its equilibrium position, we can express this harmonic position function as

$$s(x, t) = s_{\max} \cos(kx - \omega t) \quad (2.2)$$

where $s_{\max}$ is the maximum position of the element relative to equilibrium. This is often called the displacement amplitude of the wave. The parameter

k is the wave number and ω is the angular frequency of the piston. Note that the displacement of the element is along x , in the direction of propagation of the sound wave, which means we are describing a longitudinal wave.

The variation in the gas pressure ΔP measured from the equilibrium value is also periodic. For the position function in Equation 2.2, ΔP is given by

$$\Delta P = \Delta P_{\max} \sin (kx - \omega t) \quad (2.3)$$

where the pressure amplitude $\Delta P_{\max}$ —which is the maximum change in pressure from the equilibrium value—is given by

$$\Delta P_{\max} = \rho v \omega s_{\max} \quad (2.4)$$

Thus, we see that a sound wave may be considered as either a displacement wave or a pressure wave. A comparison of Equations 2.2 and 2.3 shows that the pressure wave is 90° out of phase with the displacement wave. Graphs of these functions are shown in Figure 2.3. Note that the pressure variation is a maximum when the displacement from equilibrium is zero, and the displacement from equilibrium is a maximum when the pressure variation is zero.

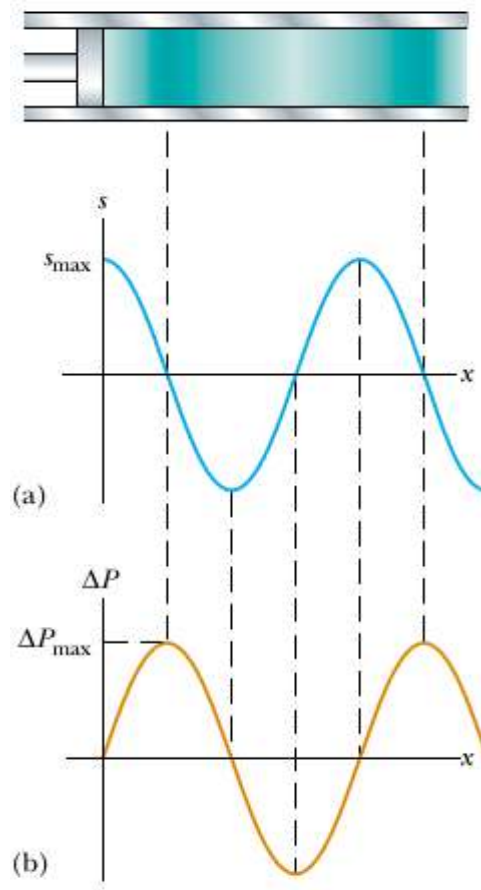


Figure 2.3 (a) Displacement amplitude and (b) pressure amplitude versus position for a sinusoidal longitudinal wave.

Example 2.2 Hearing Limits

The faintest sounds the human ear can detect at a frequency of 1000 Hz correspond to an intensity of about $1.00 \times 10^{-12} \text{ W/m}^2$ —the so-called threshold of hearing. The loudest sounds the ear can tolerate at this frequency correspond to an intensity of about 1.00 W/m^2 —the threshold of pain. Determine the pressure amplitude and displacement amplitude associated with these two limits.

Solution: First, consider the faintest sounds. Using Equation 3.6 and taking $v = 343 \text{ m/s}$ as the speed of sound waves in air and $\rho = 1.20 \text{ kg/m}^3$ as the density of air, we obtain

$$\begin{aligned}\Delta P_{\max} &= \sqrt{2\rho v I} \\ &= \sqrt{2(1.20 \text{ kg/m}^3)(343 \text{ m/s})(1.100 \times 10^{-12} \text{ W/m}^2)} \\ &= 2.87 \times 10^{-5} \text{ N/m}^2\end{aligned}$$

Because atmospheric pressure is about 10^5 N/m^2 , this result tells us that the ear is sensitive to pressure fluctuations as small as 3 parts in 10^{10} !

We can calculate the corresponding displacement amplitude by using Equation 2.4, recalling that $\omega = 2\pi f$:

$$\begin{aligned}s_{\max} &= \frac{\Delta P_{\max}}{\rho v \omega} = \frac{2.87 \times 10^{-5} \text{ N/m}^2}{(1.20 \text{ kg/m}^3)(343 \text{ m/s})(2\pi \times 1.000 \text{ Hz})} \\ &= 1.11 \times 10^{-11} \text{ m}\end{aligned}$$

This is a remarkably small number! If we compare this result for $s_{\max}$ with the size of an atom (about 10^{-10} m), we see that the ear is an extremely sensitive detector of sound waves. In a similar manner, one finds that the loudest sounds the human ear can tolerate correspond to a pressure amplitude of 28.7 N/m^2 and a displacement amplitude equal to $1.11 \times 10^{-5} \text{ m}$.

2.3 Sound Level in Decibels

Example 3.2 illustrates the wide range of intensities the human ear can detect. Because this range is so wide, it is convenient to use a logarithmic scale, where the sound level-(Greek beta) is defined by the equation

$$\beta = 10 \log \left(\frac{I}{I_0} \right) \quad (2.5)$$

The constant I_0 is the reference intensity, taken to be at the threshold of hearing ($I_0 = 1.00 \times 10^{-12} \text{ W/m}^2$), and I is the intensity in watts per square meter to which the sound level β corresponds, where β is measured in decibels(dB). On this scale, the threshold of pain ($I = 1.00 \text{ W/m}^2$) corresponds to a sound level of $\beta = 10 \log [(1 \text{ W/m}^2)/(10^{-12} \text{ W/m}^2)] = 10 \log (10^{12}) = 120 \text{ dB}$, and the threshold of hearing corresponds to $\beta = 10 \log [(10^{-12} \text{ W/m}^2)/(10^{-12} \text{ W/m}^2)] = 0 \text{ dB}$.

Prolonged exposure to high sound levels may seriously damage the ear. Ear plugs are recommended whenever sound levels exceed 90dB. Recent evidence suggests that “noise pollution” may be a contributing factor to high blood pressure, anxiety, and nervousness. Table 2.2 gives some typical sound-level values.

Table 2.2: Sound Levels

Source of Sound	β (dB)
Nearby jet airplane	150
Jackhammer; machine gun	130
Siren; rock concert	120
Subway; power mower	100
Busy traffic	80
Vacuum cleaner	70
Normal conversation	50
Mosquito buzzing	40
Whisper	30
Rustling leaves	10
Threshold of hearing	0

Example 2.4 Sound Levels

Two identical machines are positioned the same distance from a worker. The intensity of sound delivered by each machine at the location of the worker is $2.0 \times 10^{-7} \text{ W/m}^2$.

Find the sound level heard by the worker

- (A) when one machine is operating
- (B) when both machines are operating.

Solution

(A) The sound level at the location of the worker with one machine operating is calculated from Equation 3.8:

$$\beta_1 = 10 \log \left(\frac{2.0 \times 10^{-7} \text{ W/m}^2}{1.00 \times 10^{-12} \text{ W/m}^2} \right) = 10 \log(2.0 \times 10^5) = 53 \text{ dB}$$

B) When both machines are operating, the intensity is doubled to $4.0 \times 10^{-7} \text{ W/m}^2$; therefore, the sound level now is

$$\beta_2 = 10 \log \left(\frac{4.0 \times 10^{-7} \text{ W/m}^2}{1.00 \times 10^{-12} \text{ W/m}^2} \right) = 10 \log(4.0 \times 10^5) = 56 \text{ dB}$$

From these results, we see that when the intensity is doubled, the sound level increases by only 3dB.

What If? Loudness is a psychological response to a sound and depends on both the intensity and the frequency of the sound. As a rule of thumb, a doubling in loudness is approximately associated with an increase in sound level of 10dB. (Note that this rule of thumb is relatively inaccurate at very low or very high frequencies.) If the loudness of the machines in this example is to be doubled, how many machines must be running?

Answer: Using the rule of thumb, a doubling of loudness corresponds to a sound level increase of 10dB.

Thus,

$$\beta_2 - \beta_1 = 10 \text{ dB} = 10 \log \left(\frac{I_2}{I_0} \right) - 10 \log \left(\frac{I_1}{I_0} \right) = 10 \log \left(\frac{I_2}{I_1} \right)$$

$$\log \left(\frac{I_2}{I_1} \right) = 1$$

$$I_2 = 10 I_1$$

Thus, ten machines must be operating to double the loudness.

2.4 Loudness and Frequency

The discussion of sound level in decibels relates to a physical measurement of the strength of a sound. Let us now consider how we describe the psychological “measurement” of the strength of a sound.

Of course, we don’t have meters in our bodies that can read out numerical values of our reactions to stimuli. We have to “calibrate” our reactions somehow by comparing different sounds to a reference sound. However, this is not easy to accomplish. For example, earlier we mentioned that the threshold intensity is 10^{-12}W/m^2 , corresponding to an intensity level of 0dB. In reality, this value is the threshold only for a sound of frequency 1000Hz, which is a standard reference frequency in acoustics. If we perform an experiment to measure the threshold intensity at other frequencies, we find a distinct variation of this threshold as a function of frequency. For example, at 100Hz, a sound must have an intensity level of about 30dB in order to be just barely audible! Unfortunately, there is no simple relationship between physical measurements and psychological “measurements.” The 100-Hz, 30-dB sound is psychologically “equal” to the 1000-Hz, 0-dB sound (both are just barely audible) but they are not physically equal ($30\text{ dB} \neq 0\text{ dB}$).

By using test subjects, the human response to sound has been studied, and the results are shown in Figure 2.4 (the white area), along with the approximate frequency and sound-level ranges of other sound sources. The lower curve of the white area corresponds to the threshold of hearing. Its variation with frequency is clear from this diagram. Note that humans are sensitive to frequencies ranging from about 20Hz to about 20000Hz. The upper bound of the white area is the threshold of pain. Here the boundary of

the white area is straight, because the psychological response is relatively independent of frequency at this high sound level.

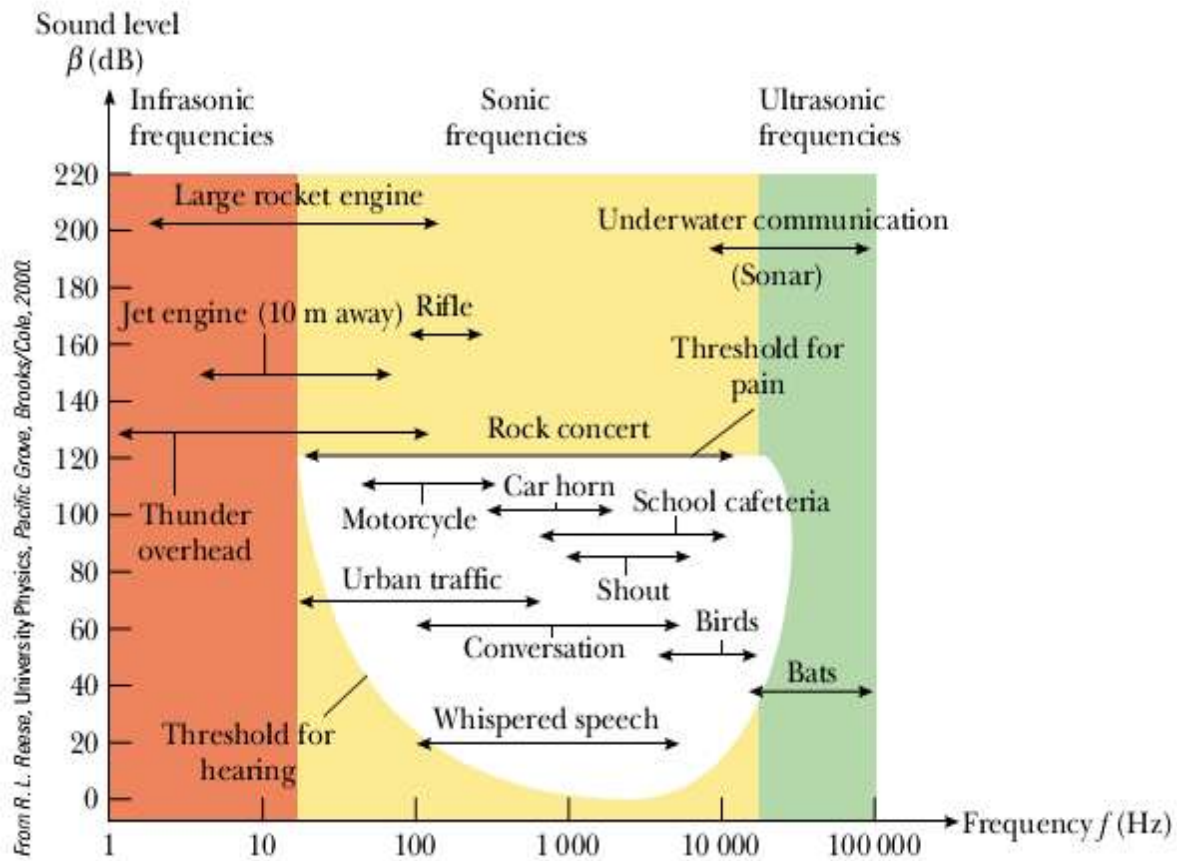


Figure 2.4 Approximate frequency and sound level ranges of various sources and that of normal human hearing, shown by the white area.

The most dramatic change with frequency is in the lower left region of the white area, for low frequencies and low intensity levels. Our ears are particularly insensitive in this region. If you are listening to your stereo and the bass (low frequencies) and treble (high frequencies) sound balanced at a high volume, try turning the volume down and listening again. You will probably notice that the bass seems weak, which is due to the insensitivity of the ear to low frequencies at low sound levels, as shown in Figure 2.4.

2.5 Standing Waves

The modes of vibration associated with resonance in extended objects like strings and air columns have characteristic patterns called standing waves. These standing wave modes arise from the combination of reflection and interference such that the reflected waves interfere constructively with the incident waves. An important part of the condition for this constructive interference for stretched strings is the fact that the waves change phase upon reflection from a fixed end. Under these conditions, the medium appears to vibrate in segments or regions and the fact that these vibrations are made up of traveling waves is not apparent - hence the term "standing wave", as shown in Figure 2.5.

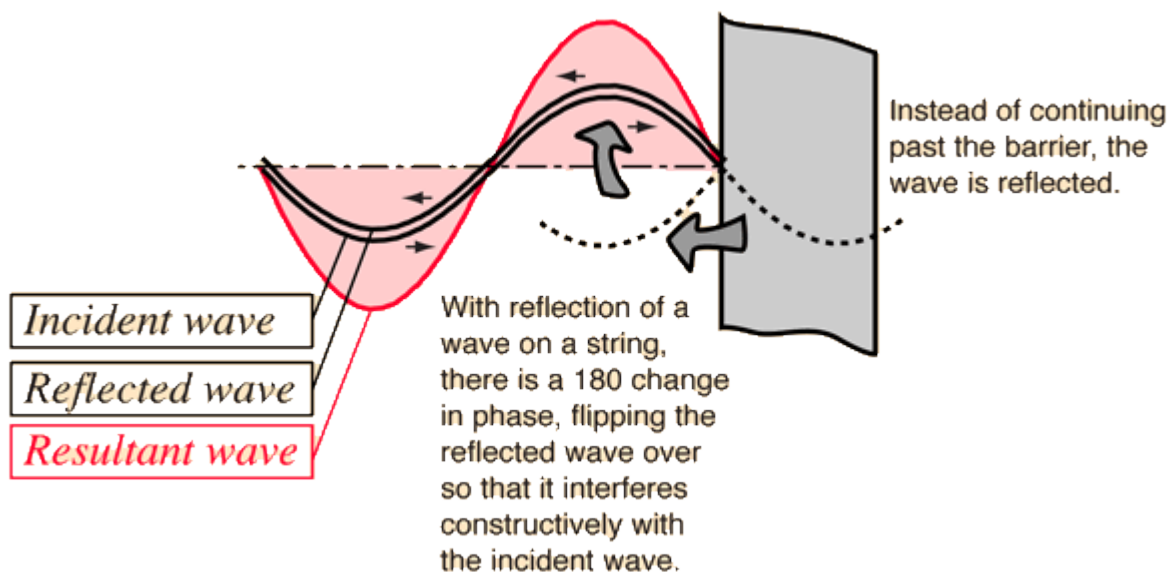


Figure 2.5 Schematic diagram express the concept of standing wave.

The behavior of the waves at the points of minimum and maximum vibrations (nodes and antinodes) contributes to the constructive interference which forms the resonant standing waves. The illustration above involves

the transverse waves on a string, but standing waves also occur with the longitudinal waves in an air column. Standing waves in air columns also form nodes and antinodes, but the phase changes involved must be separately examined for the case of air columns.

The term standing wave is often applied to a resonant mode of an extended vibrating object. The resonance is created by constructive interference of two waves which travel in opposite directions in the medium, but the visual effect is that of an entire system moving in simple harmonic motion. The sketches shown in Figure 2.6 illustrate the fundamental and second harmonic standing waves for a stretched string.

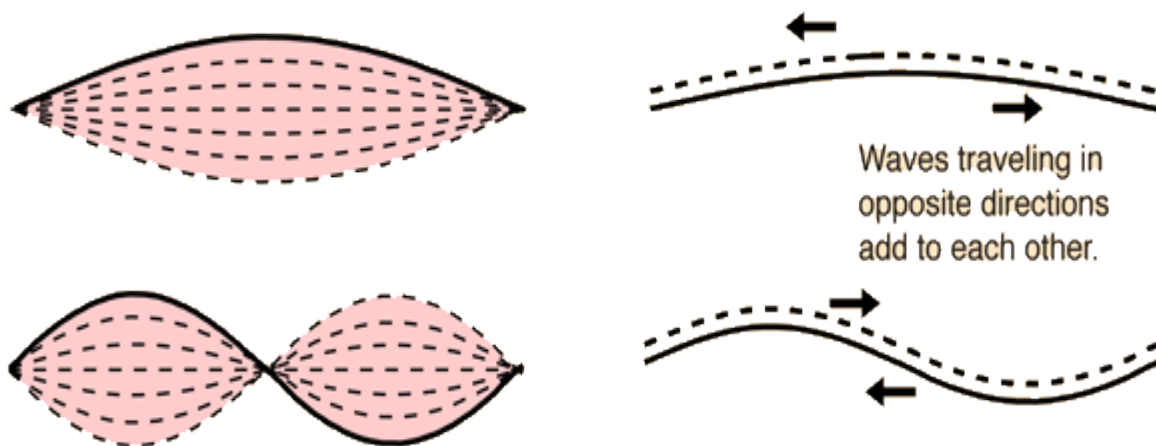
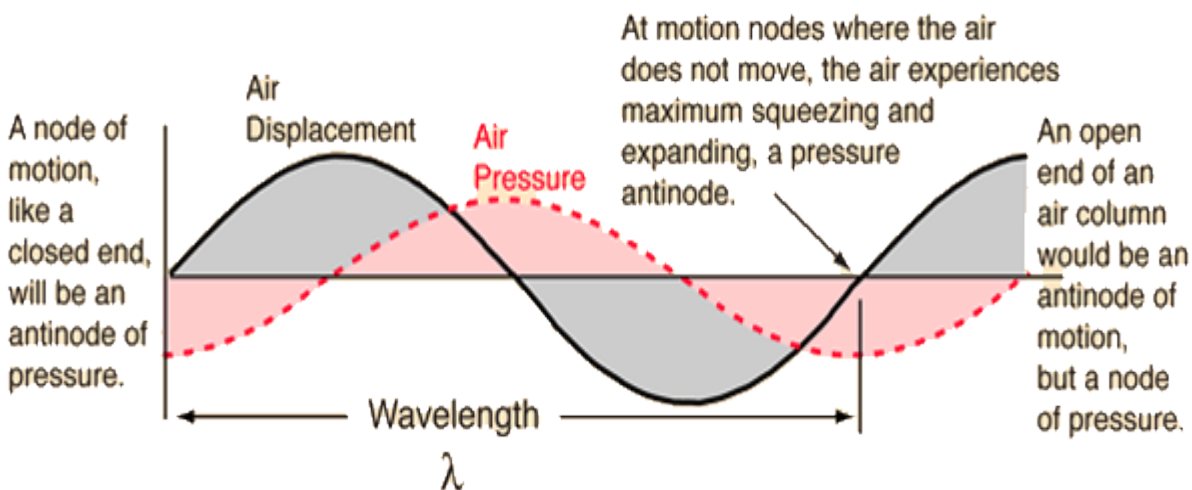


Figure 2.6 The sketches illustrate the fundamental and second harmonic standing waves for a stretched string.

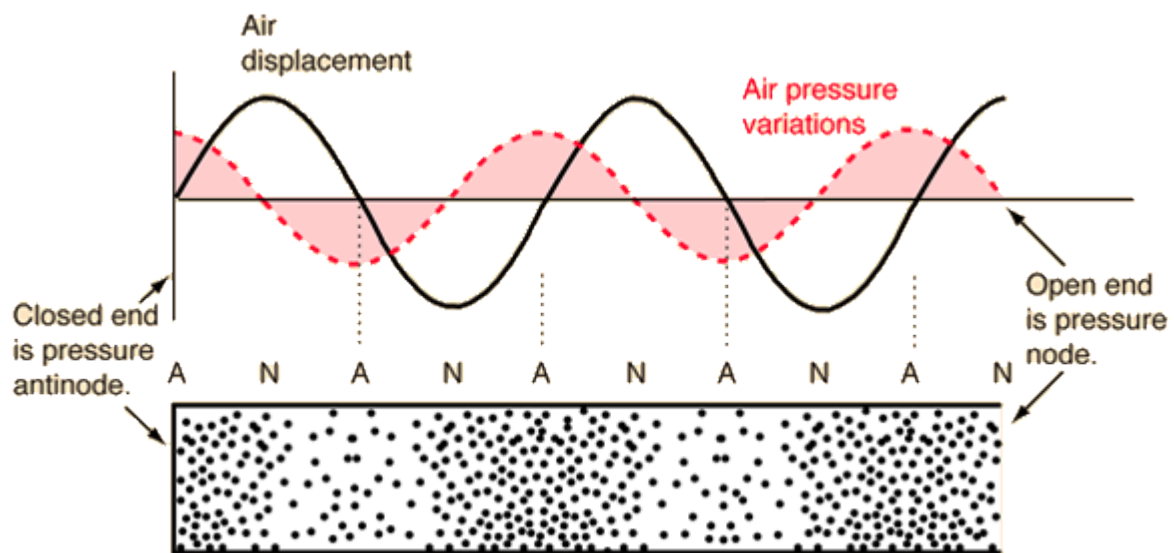
2.6 Displacement and Pressure

The standing waves associated with resonance in air columns have been discussed mainly in terms of the displacement of air in the columns. They can also be visualized in terms of the pressure variations in the column. A node for displacement is always an antinode for pressure and vice versa, as illustrated below. When the air is constrained to a node, the air motion will be alternately squeezing toward that point and expanding away from it, causing the pressure variation to be at a maximum. This view of resonant modes in terms of pressure waves makes it easier to see why the mouthpiece end of a wind instrument is a node for the resonances. For example, the clarinet is acoustically a closed-end cylindrical air column because the mouthpiece end acts as a pressure antinode. An oboe is induced to produce its upper register by opening a hole near the mouthpiece, releasing pressure to make that point a pressure node and therefore a displacement antinode.



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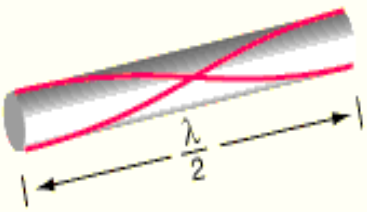
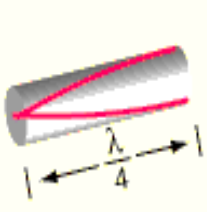
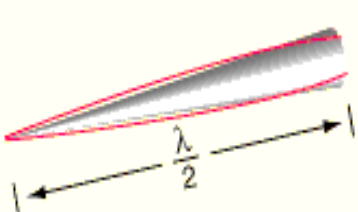
A node for displacement is always an antinode for pressure and vice versa, as illustrated below. When the air is constrained to a node, the air motion will be alternately squeezing toward that point and expanding away from it, causing the pressure variation to be at a maximum. This view of resonant modes in terms of pressure waves makes it easier to see why the mouthpiece end of a wind instrument is a node for the resonances. For example, the clarinet is acoustically a closed-end cylindrical air column because the mouthpiece end acts as a pressure antinode.



One way to demonstrate standing waves in an air column is by stroking a metal rod to set up the longitudinal standing wave in the rod. If a disc is placed on the end of the rod, it can set up standing waves in the air column. The motion of the air at the antinodes is sufficient to move cork dust to produce a pattern in the dust. The illustration at left is part of a Kundt's tube designed to produce the standing waves.

2.7 Air Column Resonance

The resonant frequencies of air columns depend upon the speed of sound in air as well as the length and geometry of the air column. Longitudinal pressure waves reflect from either closed or open ends to set up standing wave patterns. Important in the visualization of these standing waves is the location of the nodes and antinodes of pressure and displacement for the air in the columns. For temperature $T = \dots$ C the speed of sound is m/s.

		
<u>Open cylinder</u>	<u>Closed cylinder</u>	<u>Closed cone</u>
Length L = m	Length L = m	Length L = m
$f_1 =$ Hz	$f_1 =$ Hz	$f_1 =$ Hz
$f_2 =$ Hz	$f_2 = \times$ Hz	$f_2 =$ Hz
$f_3 =$ Hz	$f_3 =$ Hz	$f_3 =$ Hz
$f_4 =$ Hz	$f_4 = \times$ Hz	$f_4 =$ Hz
$f_5 =$ Hz	$f_5 =$ Hz	$f_5 =$ Hz

Any of the values may be changed. The calculation defaults to a 1 meter open column at temperature 20 C if data for length and temperature are not entered.

2.8 Open Cylinder Air Column

A cylindrical air column with both ends open will vibrate with a fundamental mode such that the air column length is one half the wavelength of the sound wave. Each end of the column must be an antinode for the air motion since the ends are open to the atmosphere and cannot produce significant pressure changes. For the fundamental mode, there is one node at the center. The basic wave relationship leads to the frequency of the fundamental:

$$f_1 = \frac{v_{\text{sound}}}{2L}$$

The open air column can produce all harmonics. Open cylinders are employed musically in the flute, the recorder, and the open organ pipe.

2.9 Harmonics of Open Air Column

An open cylindrical air column can produce all harmonics of the fundamental. The positions of the nodes and antinodes are reversed compared to those of a vibrating string, but both systems can produce all harmonics. The sinusoidal patterns indicate the displacement nodes and antinodes for the harmonics. A pressure node corresponds to a displacement antinode, and the harmonic patterns can also be visualized in terms of air pressure or density patterns.

